UNWAL PACIFIC
Foundation | IIT-JEE | Medical | Repeater

Regional Mathematical Olympiad – 2023

Time: 3 hours **October 29, 2023**

Instructions

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let N be the set of all positive integers and

Find the largest positive integer m such that m divides for all

2. Let be a semicircle with as the bounding diameter and let CD be a variable chord of the semicircle of constant length such that lie in the interior of the arc Let E be a point on the diameter such that are equally inclined to the line Prove that:

(a) The measure of is a constant.

(b) The circumcircle of triangle passes through a fixed point.

3. For any natural number n, expressed in base 10, let $s(n)$ denote the sum of all its digits. Find all natural numbers m and n such that $m < n$ and

and .

4. Let be two intersecting circles with centres respectively. Let be a line that intersects at points and at points such that are collinear in that order. Let the perpendicular bisector of segment intersect at points and the perpendicular bisector of segment intersect at point such that are on the same side of . Prove that the midpoint of and are collinear.

5. Let be positive integers. Determine all positive real numbers which satisfy

6. Consider a set of 16 points arranged in a square grid formation. Prove that if any 7 of these points are coloured blue, then there exists an isosceles right-angled triangle whose vertices are all blue.