

KVPY SA | SOLUTIONS

Part – 1: Mathematics

1.(C) Since $\lceil x^2 \rceil$ is an integer and $\lceil x^2 \rceil = x + 1$ so x is an integer.

$\Rightarrow \lceil x^2 \rceil = x^2 = x + 1$ or $x^2 - x - 1 = 0$. But it does not have any integral solution.

2.(A) $p_1(x) = x^3 - 2020x^2 + b_1x + c_1$

$$p_2(x) = x^3 - 2021x^2 + b_2x + c_2$$

$$\text{Since } p_1(x)q_1(x) + p_2(x)q_2(x) = x^2 - 3x + 2$$

$$\Rightarrow q_1(x) = 1 \text{ and } \Rightarrow q_2(x) = -1 \Rightarrow p_1(x) - p_2(x) = x^2 - 3x + 2$$

$$\text{Or } x_2 + x(b_1 - b_2) + c_1 - c_2 = x^2 - 3x + 2 \Rightarrow b_2 - b_1 = 3 \text{ as } c_1 - c_2 = 2$$

And the common roots α and β are the roots of the equation

$$x^2 - 3x + 2 = 0 \text{ i.e., } 1 \text{ and } 2.$$

$$\Rightarrow 1 - 2020 + b_1 + c_1 = 0 \text{ or } b_1 + c_1 = 2019$$

$$\text{Or } 8 - 8080 + 2b_1 + c_1 = 0 \text{ or } 2b_1 + c_1 = 8072$$

$$\Rightarrow b_1 = 6053; c_1 = -4034 \quad \Rightarrow b_2 = 6054; c_2 = -4034$$

$$\text{So, } p_1(3) + p_2(1) + 4028 = 0$$

3.(C) Consider $p = 4, q = 9, r = 25$

This is a counter example for

(A), (B) and (D)

So, (C) is the only choice that is left.

Let's prove it.

Claim:

If p, q, r are positive rationals and $\sqrt{p} + \sqrt{q} + \sqrt{r}$ is also rational, then $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are all rationals.

Let's prove the contrapositive that if at least one of $\sqrt{p}, \sqrt{q}, \sqrt{r}$ is not rational, then $\sqrt{p} + \sqrt{q} + \sqrt{r}$ is not rational.

Case I When exactly one number is irrational. WLOG say that is $\sqrt{p}$ while $\sqrt{q}$ and $\sqrt{r}$ are rational, then

$$\sqrt{p} + \sqrt{q} + \sqrt{r} = w \text{ (rational)}$$

$$\downarrow \quad \downarrow$$

Irrational Rational

Contradiction.

Case II when two numbers are irrational and one is rational.

WLOG Let $\sqrt{p}$ and $\sqrt{q}$ be irrational while $\sqrt{r}$ is rational.

$$\sqrt{p} + \sqrt{q} + \sqrt{r} = w \text{ (rational)}$$

$$\sqrt{q} = w - \sqrt{r} + \sqrt{p}$$

$$q = w^2 + r + p - 2w\sqrt{r} - 2w\sqrt{p} + 2\sqrt{r}p$$

Rational

$$\Rightarrow 2\sqrt{p}(w - \sqrt{r}) \text{ must be rational}$$

But $w - \sqrt{r}$ is rational

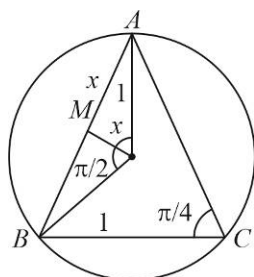
And $\sqrt{p}$ is irrational

So either $w = \sqrt{r}$ (not possible, why?)

Or we have arrived upon a contradiction

Similarly argue for the case when all of them are irrational.

4.(D)



$$x^2 + x^2 = 1$$

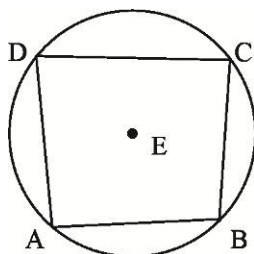
$$x = \frac{1}{\sqrt{2}} \quad AB = \sqrt{2}$$

$$5.(D) \quad AM \cdot HM \geq n^2$$

$$(x+y)\left(\frac{1}{x} + \frac{1}{y}\right) \geq 4$$

$$\frac{1}{x} + \frac{1}{y} \geq 4$$

6.(D)



Since $AE = BE = CE = DE$, therefore

ABCD is a cyclic quadrilateral with centre E.

Now, $\angle DAB + \angle BCD = 2\angle ABC$

And $\angle DAB + \angle BCD = \pi = 2\angle ABC$

$\Rightarrow \angle ABC = \pi / 2$

Since $\angle ABC$ is the middle-term of the AP hence the median.

7.(A) $2^x + 3^y = 5^{xy} = (2+3)^{xy}$

Since x and y are positive integers so $2^x \leq 2^{xy}$ and $3^y \leq 3^{xy}$ and the equality holds if $x = xy = y \Rightarrow x = y = 1$

8.(B) One digit + 2 digit number + 3 digit number

↓

$9 + 2 \times 90 + 3 \times k = 2021$

$3k = 2021 - 189$

$3k = 1832$

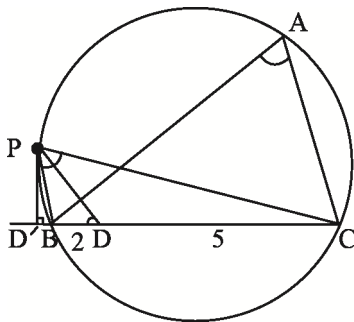
$k = \frac{1832}{3} = 610 + 2 \text{ digit}$

100, 101, 709, 710

↓

2021st digit

9.(D)



$BD : DC = 2 : 5$

$\frac{BD}{BC} = \frac{2}{7}$

Given that $\angle BAC = \angle PDB = \angle BPC$

$\angle BPC = \angle BDP$

$\angle CBP = \angle DBP$

Therefore, $\triangle PBC \sim \triangle DBP$

$$\Rightarrow \frac{ar(\triangle PBC)}{ar(\triangle DBP)} = \frac{(PC)^2}{(PD)^2} = \frac{\frac{1}{2}PD \cdot BC}{\frac{1}{2}PD \cdot BD} \Rightarrow \frac{(PD)^2}{(PC)^2} = \frac{BD}{BC} = \frac{2}{7} \Rightarrow \frac{PD}{PC} = \frac{\sqrt{2}}{\sqrt{7}}$$

10.(B)

$$2^{2018} + 1 \Bigg) 2^{\frac{2^2}{2^{2020} + 1}} \Bigg(\frac{2^{\frac{2^{2020} + 4}{-3}}}{-3}$$

$= 2^2 - 1 + 3^2 - 1 + 4^2 - 1 + 5^2 - 1 + 6^2 - 1 = 3 + 8 + 15 + 24 + 35 = 85$

11.(A) $(1 + 2020)^{2020} = 1 + (2020)^2 + 2020C_2 \cdot (2020)^2 + \dots$
 Remainder = 1

12.(A) $\triangle ABD \cong \triangle ADE$ (A, A, S)

$BD = DE$

Using angle bisector theorem

$$\frac{AD}{AC} = \frac{DE}{EC} = \frac{1}{2}$$

$AC = 2AD$

In $\triangle ACD$

So $AC^2 = AD^2 + DC^2$

$$AD = \sqrt{147}$$

$$AC = 2\sqrt{147}$$

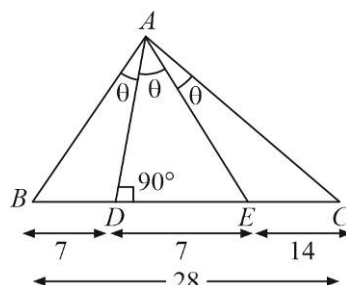
In $\triangle ADB$

$$AB^2 = AD^2 + BD^2 = 147 + 49 = 196$$

$$AB = 14$$

$$AB + AC = 38.24$$

Nearest integer = 38



13.(B) $a_1, a_2, \dots, a_5$

a_1 and a_2 are not at their respective position

= Total $-a_1$ at first $-a_2$ at first + a_1 and a_2 at first and second position = $120 - 24 - 24 + 6 = 78$

14.(B) $(n + 1)! \times m!$

15.(B) $(abcde) \times 9$

↓

a should be 1 as $9 \times 2 = 18$ (number will become 6 digit)

$$(1 bcde) \times 9 = edcb1$$

e must be $9 \times 9 = 81$

$$(1 bcd 9) \times 9 = 9 dcb 1$$

$9 \times b$ should not have any carry $b = 1$ or $b = 0$

For $b = 1$ $(11 cd 9) \times 9 = 9 dc 11$

d should be 9

$$(11 \overset{8}{c} 99) \times 9 = 99 c 11$$

↓ 91

Not possible

So $b = 0$

$$(10 c \overset{8}{d} 9) \times 9 = 9 dc 01$$

↓

$$9d + 8 \rightarrow k0$$

↓

D should be 8

$$(10 c \overset{8}{8} 9) \times 9 = 98 c 01$$

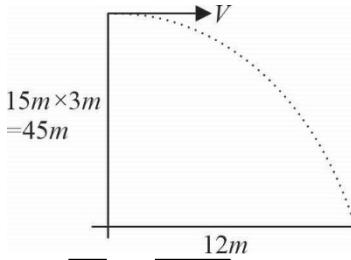
$$9c + 8 = 8c$$

$$c = 9$$

So number will be 10989

Physics

16.(D)

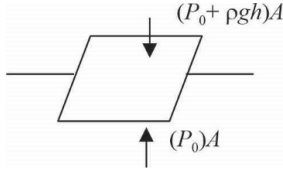


$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3 \text{ sec}$$

$$R = V \times t \Rightarrow 12 = V \times 3 \Rightarrow V = 4 \text{ m/s} = 4 \times \frac{18}{5} = 14.4 \text{ km/h}$$

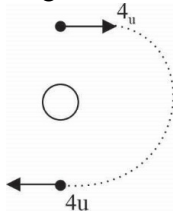
17.(A) $\sigma = \frac{F}{A} = \frac{F}{hr^2} \Rightarrow \sigma \propto \frac{1}{r^2} \therefore \frac{\sigma_1}{\sigma_2} = \left(\frac{r_2}{r_1}\right)^2 = \frac{1}{4}$

18.(A)



$$\text{Force exerted} = (\rho gh)A = (1.03 \times 10^3) \times 10 \times 100 \times (900 \times 10^{-4}) = 0.927 \times 10^5$$

19.(D) Angular momentum will remain conserved with respect to massive planet



So while exit, it has $4u$ towards $-X$ direction

With respect to planet

So, $V_{\text{aircraft wrt planet}} = -4u$

$$V_{\text{planet}} = -3u$$

$$V_{\text{aircraft}} = -7u$$

20.(B)

$$180^\circ = 180 \times 60 \times 60 = 648000 \text{ sec}$$

$$10^5 \text{ years time taken form } 648000 \text{ sec}$$

$$1 \text{ year time taken from } \frac{648000}{10^5} = 6.48 \text{ sec}$$

21.(B)



$$A_I > A_{II} \quad [\text{Given}]$$

By equation of continuity

$$A_I V_I = A_{II} V_{II} \Rightarrow V_I < V_{II}$$

Further by Bernoulli's equations

$$P_I + \frac{1}{2} \rho V_I^2 = P_{II} + \frac{1}{2} \rho V_{II}^2 \Rightarrow P_I > P_{II}$$

Pressure lower in II

22.(C) Diffraction is observed in compact disc due to concentric grooves.
So, light scattered from two grooves interfere results into diffraction.

23.(B) Range is same for complementary angles

i.e. θ and $90 - \theta$

here $\theta = 30^\circ$ so other angle will be $90 - 30 = 60^\circ$

$$h = \frac{u^2 \sin^2 30^\circ}{2g} \quad \dots(i)$$

$$h' = \frac{u^2 \sin^2 60^\circ}{2g} \quad \dots(ii)$$

Dividing equation (ii) by equation (i)

$$\frac{h'}{h} = \frac{\sin^2 60^\circ}{\sin^2 30^\circ} = 3 \Rightarrow h' = 3h$$

24.(D) COM will be initially at the centre of cylinder when empty now when completely filled it will still lie at center. In the mean while, COM shift below and then rises to come at same point.
And θ will be least for initial & final condition.

25.(D) g at height above and near to surface = g depth below surface

$$\Rightarrow g \left(1 - \frac{2h}{R} \right) = g \left(1 - \frac{d}{R} \right) \Rightarrow d = 2h = 20 \text{ km}$$

26.(B) By lens formula

$$\left[\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \right] \times u \quad \left[m = \frac{v}{u} \right] \text{ for lens}$$

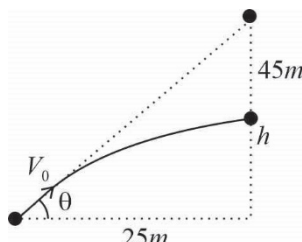
$$\Rightarrow \frac{u}{v} - 1 = \frac{u}{f} \quad \Rightarrow \quad \frac{1}{m} - 1 = \frac{u}{f}$$

From graph at $\frac{1}{m} = -200$, $u = -0.8$

Putting in equation

$$-200 - 1 = \frac{-0.8}{f} \Rightarrow f = \frac{-0.8}{-201} = 0.004$$

27.(B)



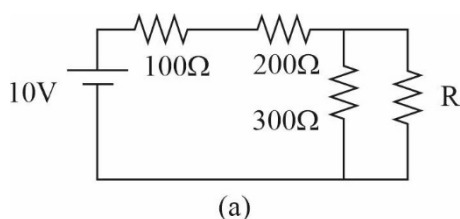
$$\tan \theta = \frac{45}{25} = \frac{9}{5}$$

By equation of trajectory let they meet at height h

$$y = x \tan \theta - \frac{gx^2(1 + \tan^2 \theta)}{2u^2}$$

$$\Rightarrow h = 5 \times \frac{9}{5} - \frac{10 \times 25^2 \left(1 + \frac{81}{25} \right)}{2 \times 24^2} \Rightarrow h = 45 - \frac{5 \times 25 \times \frac{106}{25}}{24 \times 24} = 22\text{m}$$

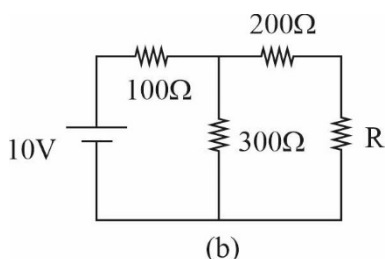
28.(A)



For the circuit (a) potential difference across R is

$$V_a = 10 - 300 \left(\frac{10}{300 + \frac{300R}{300 + R}} \right)$$

For the circuit (b) potential difference across R is



$$V_B = \frac{300}{500 + R} \times \frac{10}{100 + \frac{300(200 + R)}{500 + R}} R$$

Both of them are equal so $\frac{300}{500 + R} \times \frac{10R}{100 + \frac{300(200 + R)}{500 + R}} = 10 - 300 \left(\frac{10}{300 + \frac{300R}{300 + R}} \right)$

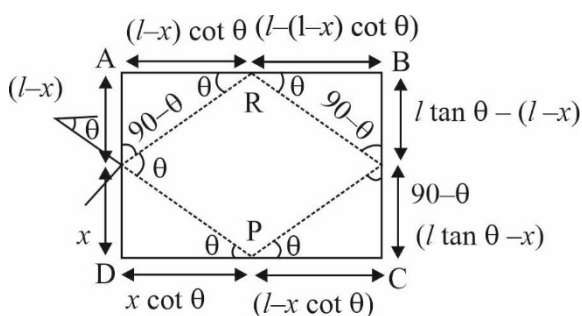
On solving we get $R = 100\Omega$

29.(C) $\frac{F_{\text{gravitational}}}{F_{\text{electrostatic}}} = \frac{\frac{G m_1 m_2}{r^2}}{\frac{K q_1 q_2}{r^2}} = \frac{6.7 \times 10^{-11} \times (9.1 \times 10^{-31})^2}{9 \times 10^9 \times (1.6 \times 10^{-19})^2} = \frac{554.827 \times 10^{-73}}{23.04 \times 10^{-29}} = 24 \times 10^{-44}$

30.(C) Let us consider edge length of square is l .

To solve the question, we will consider x as distance between opening to D (referring to the figure). Using right angle triangle, we will calculate length of BQ and QR as shown in figure.

As we know $BQ + QR = l$ we will solve it



$$l \tan \theta - x + l \tan \theta - l + x = l$$

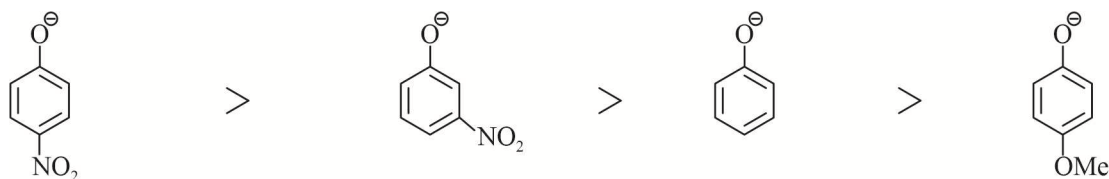
$$2 \tan \theta = 2$$

$$\boxed{\theta = 45^\circ}$$

Chemistry

31.(C) Acidic strength of compound $\propto$ stability of conjugate base.

Order of stability of conjugate base.



Both -I and -R effect of NO₂
Stabilizes oxy anion
by electron
withdrawing effect

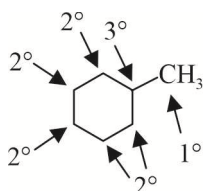
No resonance effect
from meta position
-I effect stabilizes
oxy-anion

Reference compound

-I, + R
electron donation into the ring is more
significant than its inductive electron
withdrawal from ring

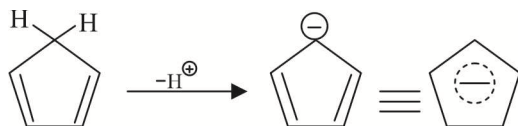
32.(A) Compound I has centre of symmetry and compound III has plane of symmetry. Hence they are optically inactive.

33.(D)



34.(B) Aniline is steam volatile and insoluble in water while all are soluble in water so can be purified by steam distillation.

35.(B)



Aromatic (6 π electrons)

Follows (4n + 2) πe^- Huckel's rule

$$4n + 2 = 6$$

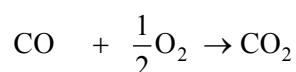
$$n = 1$$

36.(B) 1 L of CO and H₂

CO : H₂

0.5L : 0.5L

$$9 \text{ L of (20\% by volume) O}_2 = \frac{20}{100} \times 9 = 1.8 \text{ L of O}_2$$



0.5 L 1.8 L 0
0 1.3 L 0.5 L

CO is limiting reagent

$$0.5 \text{ L of CO}_2 \text{ at STP} = \frac{0.5}{22.4} \text{ moles of CO}_2 = 0.022 \text{ moles of CO}_2.$$

37.(C) Work function = 2 eV .

$$E_1 = \frac{1240}{400} = 3.1 \text{ eV}$$

$$E_2 = \frac{1240}{800} = 1.5 \text{ eV (Not sufficient to eject the electrons).}$$

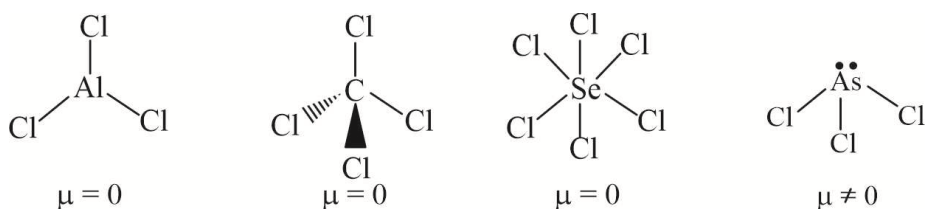
38.(D) Equilibrium can be achieved from either side of the reaction.

$$39.(D) \quad V_{MP} : \bar{V} : V_{RMS} \\ 1 : 1.125 : 1.224$$

40.(C) Electronic configuration of Cu is $3d^{10} 4s^1$

$$4s^1 \Rightarrow n = 4, l = 0, m = 0, s = +\frac{1}{2} \text{ or } -\frac{1}{2}$$

41.(D)



42.(B) Smaller the size of cation, higher the polarizing power and more the covalent character.

$$\text{Covalent character} \propto \text{Polarising power of cation} \propto \frac{\text{Charge on cation}}{\text{Size of cation}}$$

Size of cation

$$\text{Ba}^{+2} > \text{Sr}^{+2} > \text{Ca}^{+2} > \text{Mg}^{+2}$$

43.(C) 2.005 four significant figures.

44.(A) In path AB $\rightarrow$ Isothermal expansion

So T constant. $\Delta T = 0$

$$\Delta U \propto \Delta T = 0$$

So $\Delta U = q + W$

$$q = -w > 0 \quad (w_{\text{expansion}} < 0)$$

$$\text{So } \Delta S_{\text{sys}} = \frac{q}{T} > 0$$

So in path AB $\rightarrow$ Temperature remains same and entropy increases

In path BC $\rightarrow$ Adiabatic expansion

$$q = 0$$

$$\text{So } \Delta S_{\text{sys}} = \frac{q}{T} = 0$$

$$\Delta U = q + w = \Delta U = w < 0 \quad (w_{\text{expansion}} < 0)$$

So $\Delta U \propto \Delta T < 0$

Temperature decreases and entropy is constant.

In path CD → Isothermal compression

So T constant

$$\Delta U \propto \Delta T = 0$$

$$\Delta U = q + w$$

$$q = -w < 0 \quad w_{\text{compression}} > 0$$

$$\text{So } \Delta S_{\text{sys}} = \frac{q}{T} < 0$$

So in path CD → Temperature remains same and entropy decreases

In path DA → Adiabatic compression

$$q = 0$$

$$\text{So } \Delta S_{\text{sys}} = \frac{q}{T} = 0$$

$$\Delta U = q + w = \Delta U = w > 0 \quad (w_{\text{compression}} > 0)$$

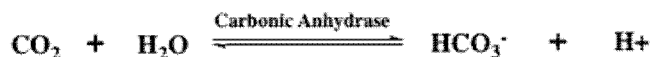
So $\Delta U \propto \Delta T > 0$

Temperature increase and entropy is constant.

- 45.(C)** On going down the group, size increases hence ionization potential decreases so potassium (K) will have lesser ionisation potential than sodium (Na).

Biology

- 46.(B)** Correct answer is “(b) CO_2 ”, because carbonic anhydrase combines carbon dioxide with water to yield carbonic acid that spontaneously dissociates into hydrogen ions and bicarbonate.



- 47.(D)** The correct answer is “(d) Acidification of ingested food”, because the intestinal juice (succus entericus) is alkaline and neutralizes the acidity of the chyme entering the duodenum.
- 48.(B)** The correct answer is “(b) Glycogen” as insulin causes the conversion of excess blood glucose to glycogen in liver and muscles by triggering the process of glycogenesis.
- 49.(A)** The metabolic requirement of poikilotherms (cold blooded org.) are less than that of homeotherms (warms blooded org.) As homeotherms need more energy to maintain their body temperature / osmotic concentration constant w.r.t external conditions
- 50.(C)** During Aerobic respiration in ETS (Electron Transport system) pumping of proton occurs across inner mitochondrial membrane.
- 51.(A)** Haemophilia – x- linked recessive
⇒ Phenylketonuria, Sickle cell anemia & B-thalassemia are autosomal disorders.
- 52.(D)** Fruit is ripened ovary and seed is ripened ovule in angiosperms
- 53.(C)** The correct answer is “(c) Milkmaids previously infected with cowpox did not contract small pox”, because Edward Jenner scraped pus from cowpox blisters on the hands of Sarah Nelmes, a milkmaid who had caught cowpox from a cow called Blossom and inoculated James Phipps, an 8 year old boy, with it. After developing mild symptoms of cowpox the boy recovered and didn't catch small pox. This was

because of the structural commonality in the Variola viruses that cause the two diseases. This structural similarity made the adaptive immune system of the boy 'learn' to fight the antigens that were common to the pathogens of the two diseases.

54.(A) $AABBCC \times aabbcc$ (Parents)

↓

$AaBbcc$ (f_1 plants)

Number of possible from (f_1 plants) genotypes 2^n ($n=3$) i.e. 2^3 or 8 types

55.(D) The correct answer is "d) Prophase, Metaphase Anaphase, Telophase.

56.(C) The correct answer is "c) Residual volume", which is what we call the amount of air left in lungs after full and forced expiration; amounts to 1.1-1.2 L in women and men respectively and is measured by a Spirometer.

57.(B) The correct answer is "b) P(ii) Q(iv) R(i) S(iii)" because Radula is found in the buccal cavity of gastropod molluscs, true metameric segmentation is a characteristic feature of phylum annelida, Pseudocoelom is the false body cavity found in nematode worms and echinoderms display a radial symmetry in adults.

58.(A) The correct answer is "a) Alfred Russell Wallace", because once he had his theory, Darwin was meticulous about gathering and refining evidence before making his idea public. He was in the process of writing his "big book" to present his research when the naturalist Alfred Russel Wallace independently conceived of the principle and described it in an essay that he sent to Darwin to forward to Charles Lyell. Lyell and Joseph Dalton Hooker decided to present his essay together with unpublished writings that Darwin had sent to fellow naturalists, and On the Tendency of Species to form Varieties; and on the Perpetuation of Varieties and Species by Natural Means of Selection was read to the Linnean Society of London announcing co-discovery of the principle in July 1858.

59.(C) Maximum concentration of harmful chemicals is found in top carnivore of a food chain (Biomagnification)

60.(D) The correct answer is d) Single stranded RNA" because all coronaviruses are single stranded retroviruses with unique spike proteins on the protein envelope.

Part – 2: Mathematics

61.(C) Number with all even digits and not divisible by 4 = 16

Number with all odd _____ = $(5)^4$
= 625

$$\text{So, } P(E) = \frac{n(E)}{n(S)} = \frac{16}{641}$$

62.(C) $\frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC}$

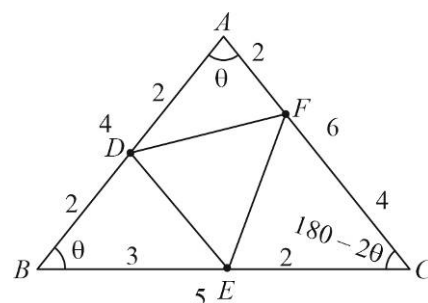
$$\text{Area of } \triangle BED = \frac{1}{2} \sin B : 2 \times 3 = 3 \sin B = \frac{3}{10} \text{ area of } \triangle ABC$$

$$\text{Area of } \triangle ADF = \frac{1}{2} \sin A \times 2 \times 2 = 2 \sin A = \frac{1}{6} \text{ area of } \triangle ABC$$

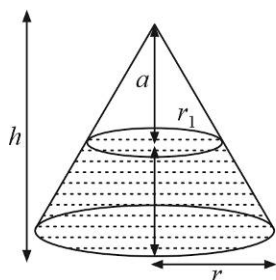
$$\text{Area of } \triangle CEF = \frac{1}{2} \times \sin C \times 2 \times 4 = 4 \sin C = \frac{4}{15} \text{ area of } \triangle ABC$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times 4 \times 6 \times \sin A = \frac{1}{2} \times 4 \times 5 \sin B = \frac{1}{2} \times 5 \times 6 \times \sin C \\ &= 12 \sin A = 10 \sin B = 15 \sin C \end{aligned}$$

$$\text{Area of } \triangle DEF = 1 - \left(\frac{3}{10} + \frac{1}{6} + \frac{4}{15} \right) = \frac{30 - 9 - 5 - 8}{30} = \frac{8}{30} = \frac{4}{15} \text{ area of } \triangle ABC$$



64.(B)



$$\frac{r_1}{a} = \frac{r}{h}$$

$$r_1 = \frac{a}{h}r$$

volume left

$$= \frac{1}{3}\pi r_1^2 \cdot a$$

$$v_1 = \frac{1}{3}\pi \frac{a^3 r^2}{h^2}$$

$$v_1 = v_2$$

$$\frac{4a^2}{h^2} = 3 + \frac{a^2}{16h^2} - \frac{3a}{4h}$$

$$\frac{a}{h} = t$$

$$4t^2 = 3 + \frac{t^2}{16} - \frac{3}{4}t$$

$$64t^2 = 48 + t^2 - 12t$$

$$63t^2 + 12t - 48 = 0$$

$$21t^2 + 4t - 16 = 0$$

$$t = \frac{-4 \pm \sqrt{16 + 16 \times 84}}{2 + 21} = \frac{-4 + 4\sqrt{85}}{2 \times 21}$$

$$\frac{h}{a} = \frac{21}{2\sqrt{85} - 2} = \frac{\sqrt{85} + 1}{8}$$

65.(D) m should divide $(m+1)!$

But 4 does not divide 3!

$$n^3 + 2n^2 + n = n(n^2 + 2n + 1)$$

↓

$$n(n+1)^2 \mid n! \text{ or } (n+1)^2 \mid (n-1)!$$

$$(n+1)^2 \mid (n-1)!$$

$$\text{Let } n = \underbrace{99\dots9}_{k \text{ times}} \text{ then } n+1 = \underbrace{1000\dots0}_{k \text{ times}}$$

Notice $(n+1)^2 = (10)^{2k}$ and the exponent of 10 in $(n-1)!$

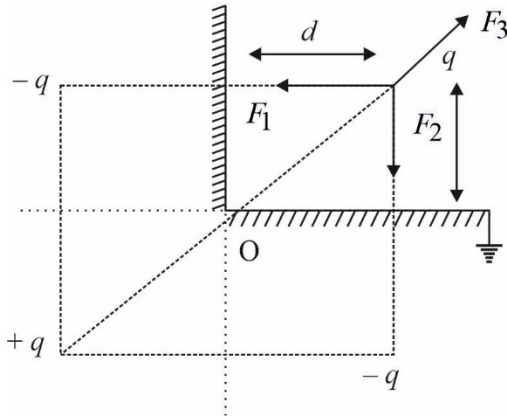
$$\left[\frac{999\dots8}{10} \right] + \left[\frac{999\dots8}{10^2} \right] + \dots + (10^{k-1} - 1) + (10^{k-2} - 1) + (10^{k-3} - 1) + \dots > 2k$$

Physics

66.(C) Using Image theory the reflection of charges will look according to shown in the figure the value of

$$F_1 = F_2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{4d^2} \text{ and the value of } F_3 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{8d^2} \text{ so resultant of all the 3 forces will be}$$

$$F_{\text{net}} = (F_1 + F_2)\cos 45^\circ - F_3 = \frac{\sqrt{2}q^2}{16\pi\epsilon_0 d^2} - \frac{q^2}{32\pi\epsilon_0 d^2}$$



$$= \frac{q^2}{32\pi\epsilon_0 d^2} (2\sqrt{2} - 1) \text{ towards O}$$

67.(B) 3 of the options are in appropriate so the question will be disqualified.

Horizontal velocity of A and B is always same and the horizontal displacement of both the them are also same so time taken by A and B will also be same.

$$t_A = t_B$$

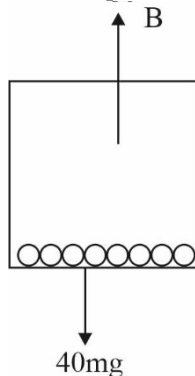
because t_A starts when A leaves horizontal position and C is also released at that moment so both of them will not have any vertical velocity so time taken by A and C to reach x will also be same

$$t_A = t_B = \sqrt{\frac{2h}{g}}$$

So from both the equations we can say $t_A = t_B = t_C$

68.(A) Because the images are projected on screen therefore they are real and real image are inverted so those who are above principle axis will go below principle axis and vice versa. Image of left object will move to right and vice versa. So option (A) is correct.

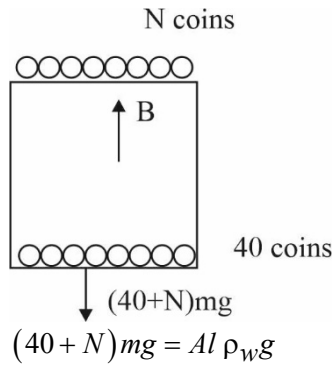
69.(B) Let us say mass of one coin is m and cross section area of cylindrical container is A. Initially the container is in equilibrium. So



$$40mg = A(0.03)\rho_w g \quad \dots(1)$$

In the next case the coins are kept on the lid.

So referring to the diagram let l length of the container is in the fluid we can say that

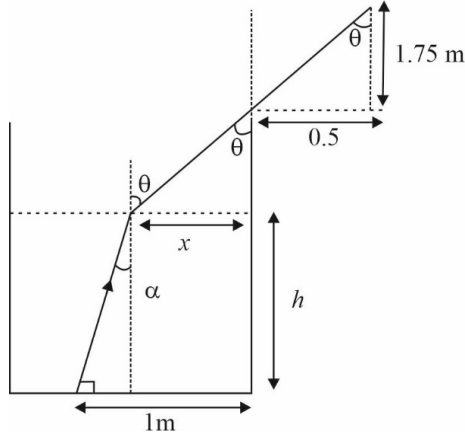


and centre of mass of the coin system will be at $d_{cm} = \frac{N \times 0.09}{40 + N}$ from bottom according to the question if

$$d_{cm} \geq \frac{l}{2} \text{ it will topple so } \frac{N \times 0.09}{40 + N} \geq \frac{(40 + N)mg}{A \rho_w g} \times \frac{1}{2} \text{ from eq. (1) } \frac{mg}{A \rho_w g} = \frac{0.03}{40}$$

$$\text{so } \frac{N \times 0.09}{40 + N} \geq \frac{(40 + N) \times 0.03}{40} \times \frac{1}{2} \text{ on solving } N \geq 10.72 \text{ so around 10 coins we can put}$$

70.(C) Referring to the figure let height of fluid is h .



$$\text{We can say } \tan \theta = \frac{0.5}{1.75} = \frac{2}{7} = \frac{x}{4-h} \quad \dots(1)$$

$$\text{and } \tan \alpha = \frac{1-x}{h}$$

from snells law we can say $\mu \sin \alpha = \sin \theta$

$$\frac{4}{3} \times \frac{1-x}{\sqrt{(1-x)^2 + h^2}} = \frac{2}{\sqrt{53}} \left(\mu = 1.33 = \frac{4}{3} \right)$$

$$\text{We can replace } x = \frac{2}{7}(4-h) \text{ from eq. (1)}$$

$$\text{So } \frac{1 - \frac{2}{7}(4-h)}{\sqrt{\left(1 - \frac{2}{7}(4-h)\right)^2 + h^2}} = \frac{6}{4\sqrt{53}}$$

On solving and taking approximation $h \simeq 2m$

$$\text{So total volume of liquid at this moment } V = \pi R^2 h = 3.14 \times 1^2 \times 2 = 6.28 m^3$$

$$\text{So time taken } t = \frac{V}{Q} = 62.8 \text{ sec.}$$

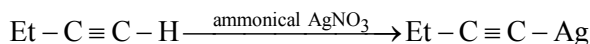
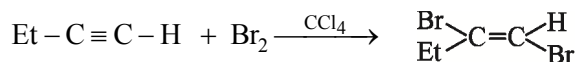
$$\simeq 63 \text{ sec.}$$

Chemistry

71.(D) C_4H_6

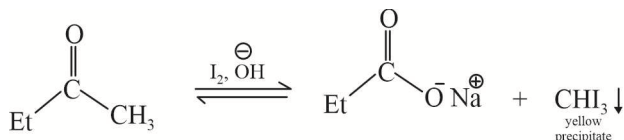
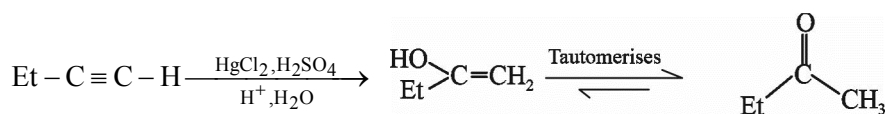
$$\text{Degree of unsaturation} = \frac{2(4+1)-6}{2} = 2$$

(A) decolourises bromine water confirms presence of unsaturation (carbon-carbon π -bond in this case.)
It also forms precipitate with $AgNO_3$ which confirms presence of terminal alkyne.

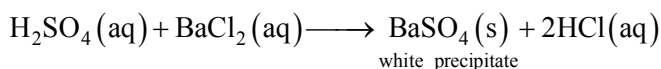


Silver alkynide (white precipitate)

On reaction with $HgCl_2, H_2SO_4$ it produce methyl ketone



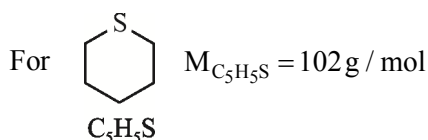
72.(D) Organic compound containing sulphur on treating with fuming HNO_3 produces H_2SO_4



$$\text{Moles of } BaSO_4 \text{ precipitated} = \frac{m_{BaSO_4}}{M_{BaSO_4}} = \frac{0.233g}{233g/mol} = 10^{-3} \text{ moles}$$

$$\text{Moles of } BaSO_4 \text{ precipitated} = \text{Moles of } H_2SO_4 \text{ formed} = \text{Moles of organic compound} = 10^{-3} \text{ moles}$$

$$\text{Molar mass of organic compound} = \frac{\text{Mass}}{\text{No. of moles}} = \frac{0.102}{10^{-3}} = 102 \text{ g/mol}$$



73.(A) $2 \rightarrow$ solute

$1 \rightarrow$ solvent

$$\frac{n_2}{n_1} = \frac{\text{Molality} \times M_1}{1000}$$

$$\frac{w_2}{w_1} \times \frac{M_1}{M_2} = \frac{\text{Molality} \times M_1}{1000}$$

$$\frac{w_2}{w_1} = \frac{\text{Molality} \times M_2}{1000} = \frac{1 \times 58}{1000} = 0.058$$

$$w_{\text{solution}} = w_1 + w_2 = (1 + 0.058)w_1 = 10\text{ g}$$

$$w_1 = \frac{10}{1.058} = 9.45\text{ g}, w_2 = 10 - 9.45 = 0.55\text{ g}$$

$$q = m \downarrow \quad C \downarrow \quad \Delta T \searrow$$

Mass of substance specific heat capacity change in temperature

$q_{\text{total}} = \text{heat absorbed by } \text{H}_2\text{O} + \text{heat absorbed by substance}$

$$= m_1 C_1 \Delta T + m_2 C_2 \Delta T = (0.55 \times 0.86 + 9.45 \times 4.2)(310 - 300) \text{ J}$$

$$= (0.473 + 39.69) \times 10 \text{ J} = 40.17 \times 10 \text{ J} = 401.7 \text{ J}$$

74.(C) Volume strength = $5.6 \times \text{Normality} = 5.6 \times 1.79 = 10$ volume H_2O_2 solution.

75.(D) Critical temperature is the highest temperature at which a gas can be liquefied applying critical pressure.

T_1 is higher than T_c so gas cannot be liquefied at this temperature.

At temperature T_2 , liquid starts appearing at point B and continues to liquefy till point A at constant pressure. A is the last point at which gaseous state is present, slight increase of pressure at constant temperature shift graph upward to complete liquid.

Biology

76.(B) The correct answer is “b) human activity significantly influenced the climate and environment”, because the influence that human existence has had on all life forms on the planet and the planet itself are considered significant enough to consider Anthropocene as the latest epoch in the evolutionary/geological time scale. The start date for this epoch is yet to be determined but the consensus is in favour of the detonation of the first atomic bomb in 1945.

77.(C) The correct answer is “c) P(iv) Q(iii) R(ii) S(v)”.

78.(B) The correct answer is “b) 7200”, because IRV per breath = $40 \times 50 = 2000$ ml, ERV per breath = $15 \times 50 = 750$ ml, Vital capacity per breath = $60 \times 50 = 3000$ ml. Thus, the tidal volume would be Vital capacity - IRV + ERV, that is, $3000 - (2000 + 750) = 250$ ml per breath. At the rate of 20 breaths per minute, the organism is circulating $250 \times 20 = 5000$ ml air in one minute. Thus, the total volume of air displaced in 24 hours would be $5000 \times 60 \times 24 = 72,00,000$ ml. Therefore, the volume in litre would be 7200 L.

79.(D) Long haired genotype = ss

Short haired $n = Ss$ or SS possible genotype of parents (i) $Ss \times Ss$

(ii) $sS \times Ss$

So that both type of progenies possible.

80.(C) The correct answer is “c) 0.108”, because the probability of a person having the disease ($10\% = 0.01$) AND the test identifying it correctly (0.99) would be $0.99 \times 0.1 = 0.099$. Additionally, the probability of a person NOT having the disease ($90\% = 0.9$) and yet the test giving a false positive result ($1 - 0.99 = 0.01$) would be $0.9 \times 0.01 = 0.009$. Thus, the total probability of a random person giving a positive result in the test would be $0.099 + 0.009 = 0.108$.