UNWAL PACIFIC
Foundation | IIT-JEE | Medical | Repeater

SOLUTIONS

Joint Entrance Exam | IITJEE-2023

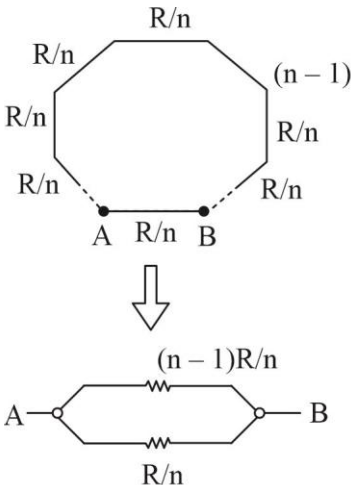
01st FEB 2023 | Evening Shift

PHYSICS

SECTION - 1

1.(4) By absubtly changing the direction of magnetic field, the flux will be negative so flux will be changed if sign is considered.

2.(3)



$$Req = \frac{(n-1)(R/n \times R/n)}{(n-1)R/n + R/n}$$

$$Req = \frac{(n-1)R^2}{n^2 R} = \frac{(n-1)R}{n^2}$$

3.(4) $T = 2\pi\sqrt{\frac{l}{g}}$

$$l = \frac{T^2 g}{4\pi^2}$$

linear relation between T^2 & l so graph will be a straight line passing through origin.

4.(4) $\frac{1}{2}$ refer to theory

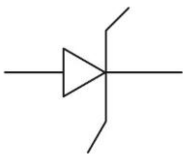
As half the energy is stored in electric field ratio of average electrical energy to the energy is half.

5.(1) $V_e = \sqrt{2gR}$

$$\therefore \frac{V_A}{V_B} = \frac{\sqrt{2g_A R_A}}{2g_B R_B} = \frac{1}{2}$$

$$\frac{g_A}{g_B} = \frac{1}{4} \times \frac{R_B}{R_A} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$$

6.(4) Refer to theory



Zener diode is a highly doped *p-n* Junction diode which works as voltage regulator in reverse Bias.

7.(4) Area of force and time graph gives impulse.

8.(4) $|f| = \frac{R}{2} = 20\ cm$

Using mirror formula

$$v = \frac{uf}{u-f}$$

for $u_1 = 15cm,\ v_1 = \frac{(-15)(-20)}{-15-(-20)} = +60cm$

for $u_2 = 25cm,\ v_2 = \frac{(-25)(-20)}{-25-(-20)} = -100cm$

distance between images $= |v_1| + |v_2| = 160\ cm$

9.(2) B at centre $= \frac{\mu_0 I \times \theta}{4\pi R}$

for semicircle $\theta = \pi$

$$B = \frac{\mu_0 I}{4R} = \frac{4\pi \times 10^{-7} \times 3}{4 \times \frac{\pi}{10}} = 3\mu T$$

10.(4) As charge always resides on surface of conductor so for same potential and radius, charge will be same for both sphere so Assertion is false but reason in correct as capacitance of sphere is $4\pi \in_0 R$

11.(2) To have minimum error in measuring voltage it is advised to use high resistance voltmeter so assertion is wrong where as reason in correct.

12.(2) $\eta = 1 - \frac{T_2}{T_1} = \frac{1}{3}$ ...(1)

$\eta' = 1 - \frac{(T_2 + x)}{T_1} = \frac{1}{6}$ ...(2)

and $T_1 = 99 + 273 = 372k$

$$1 - \frac{T_2}{372} = \frac{1}{3}$$
$$T_2 = 248k$$

On solving $x = 62k$

13.(4) $\frac{1}{2}mv_1^2 = h(2f_0) - hf_0$ ...(1)

$\frac{1}{2}mv_2^2 = h(5f_0) - hf_0$ ...(2)

solving $\frac{v_1^2}{v_2^2} = \frac{1}{4}$

$\therefore \frac{v_1}{v_2} = \frac{1}{2}$

14.(2) Velocity at highest point is not zero.

15.(1) $h = [M^1 L^2 T^{-1}] = [energy] \times [Time]$

$$G = [M^{-1} L^3 T^{-2}] = \frac{[force] \times [distance]^2}{[Mass]^2}$$

$$C = [LT^{-1}]$$

$$[h^x G^y C^z] = [M^1 L^0 T^0]$$

$$[M^1 L^2 T^{-1}]^x [M^{-1} L^3 T^{-2}]^y [LT^{-1}]^z = M^1 L^0 T^0$$

$$[M^x L^2 T^{-x}] \times [M^{-y} L^{3y} T^{-2y}] \times [L^z T^{-z}] = M^1 L^0 T^0$$

$$M^{x-y} L^{2x+3y+z} T^{-x} - 2y - z = M^1 L^0 T^0$$

$x - y = 1$ ...(1)

$2x + 3y + z = 0$ ...(2)

$-x - 2y - z = 0$ ...(3)

(2) + (3)

$x + y = 0$

$x = -y$

$x + x = 1$

$x = \frac{1}{2}$

$y = \frac{-1}{2}$

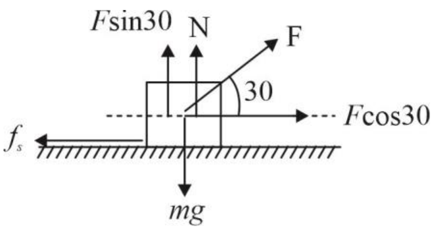
$z = \frac{1}{2}$

$$[M] = \left[h^{\frac{1}{2}} G^{-\frac{1}{2}} C^{\frac{1}{2}}\right]$$

16.(3) For ideal gas T = 0*k* or –273°C, P = 0

$Pv = nRT$ as molecules have no thermal energy they will not exert any force or pressure on walls due to collision at absolute zero temperature

17.(4) FBD of block



$\therefore$ vertical forces are balanced

$N + F\sin 30 = mg$

$N = mg - \frac{F}{2}$ ...(1)

Horizontal force

$F\cos 30 = F_{s\max} = \mu_s N$

$\therefore \frac{F\sqrt{3}}{2} = 0.25\left(mg - \frac{F}{2}\right)$

$\therefore$ solving for F

$F = 25.2N$

18.(3) modulation Index $= \frac{A_m}{A_C}$

Index $\alpha = \frac{1}{A_C}$

$\frac{I_1}{I_2} = \frac{A_{C_2}}{A_{C_1}} = \frac{2y}{y} = 2:1$

19.(4) Using Hooke's low and assuming wire to be massless

$\therefore \frac{F}{A} = \gamma\frac{\Delta L}{L}$

$\therefore \Delta L = \frac{FL}{yA} = \frac{M\frac{g}{4}\times 6}{2\times 10^{11}\times(1\times 10^{-4})}$

On solving $\Delta L = 10^{-4} = 0.1\ mm$

20.(2) $|\Delta E| = 13.6\times z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 13.6\times(4)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

on solving $|\Delta E| = 40.8\ ev$

SECTION – 2

21.(4) Optical path difference $= (\mu - 1)t$

$(\Delta x)_{optical} = (1.2 - 1)\times 10\times 10^{-6} = 2\times 10^{-6}m$

$\therefore$ net paths difference $(\Delta x) = \frac{dy}{D} - 2\times 10^{-6}$

for zero order maxima $\Delta x = 0$

$\therefore \frac{dy}{D} - 2\times 10^{-6} = 0$

$\frac{d}{D}\times x\beta_0 = 2\times 10^{-6}$

$$\therefore \frac{d}{D} \times x \times \frac{\lambda D}{d} = 2 \times 10^{-6}$$

$$x = \frac{2 \times 10^{-6}}{500 \times 10^{-9}} = \frac{2}{5} \times 10 = 4$$

22.(6) Binding energy of A = no of nucleons × B.E per nucleon

$$B.E_A = 34 \times 1.2 = 40.8 eV$$
$$B.E_B = 26 \times 1.8 = 46.8 eV$$

$\therefore$ Difference $\Delta BE = 46.8 - 40.8 = 6 eV$

23.(200)

for 1st case $0 = u^2 - 2a(S)$

$\therefore 0 = (20)^2 - 2a(500)$ ....(1)

for IInd case $(\sqrt{x})^2 = (20)^2 - 2a(250)$ .....(2)

solving eq 1 and 2

$\therefore x = 200$

24.(132)

$$\int dw = \int_{y=2}^{y=5} \vec{F} . \overrightarrow{dy}$$

$$\Rightarrow \int_2^5 (5 + 3y^2) dy = [5y]_2^5 + [y^3]_2^5$$

on solving $W = 132J$

25.(2) By continuity equation

$A \frac{dh}{dt} = av$ where v in the velocity of efflux

$$\therefore (750) \times \frac{dh}{dt} = 500 \times 10^{-2} \times 30$$

$$\therefore (750) \times x \times 10^{-3} \times 10^2 = 500 \times 10^{-2} \times 30$$

$$\Rightarrow x = 2$$

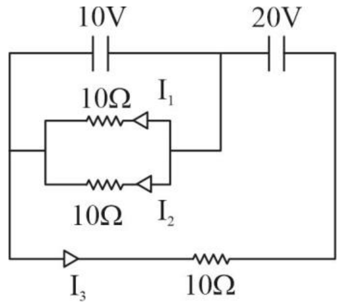
26.(3) Using parallel axis theorem

$$\therefore I = I_{cm} + M(R)^2$$

$$I = \frac{MR^2}{2} + MR^2 = \frac{3}{2} MR^2$$

$$\therefore x = 3$$

27.(2)



Using Ohm's law and Kirchhoff's voltage law

$$I_1 = \frac{10}{10} = 1 Amp,\ I_2 = \frac{10}{10} = 1\ Amp$$

$$I_3 = \frac{20-10}{10} = 1 Amp$$

$$\therefore \left| \frac{I_1 + I_3}{I_2} \right| = \frac{1+1}{1} = 2$$

28.(288)

As the electric field in only x-axis hence flux will only be associated with faces parallel to *y*-*z* plane

Net flux $\phi_{net} = \phi_1 + \phi_2$

$$= \vec{E}_1.(a^2\hat{i}) + \vec{E}_2.(a^2\hat{i}) = 0.(a^2\hat{i}) + E_0 a(a^2)$$

$$\phi_{net} = E_0 a^3$$

Using gauss's law

$$\phi_{net} = \frac{q_{enclosed}}{\in_0}$$

$$\in_0 E_0 a^3 = q_{enclosed} = Q \times 10^{-14}$$

putting the values and solving $Q = 288$

29.(44)

Induced EMF

$$E_{induced} = NBA\omega \sin(\omega t)$$

$$= 600 \times 0.4 \times (70 \times 10^{-4}) \times \frac{2\pi \times 500 \sin(30)}{60} = 44V$$

30.(67)

$$\frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{A}{2}\right)^2 + 0.25$$

$$\frac{3}{8}kA^2 = 0.25$$

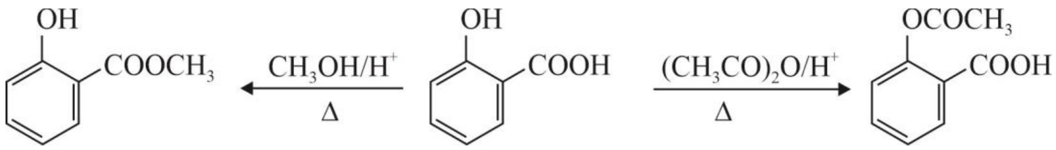
$$\frac{3}{8}k\frac{1}{100} = 0.25$$

$$k = \frac{200}{3} = 67$$

CHEMISTRY

SECTION - 1

1.(3) In statement correct option should be mentioned.

2.(3) 

Hence Y is $CH_3OH / H^+, \Delta$

X is $(CH_3CO)_2O / H^+$

3.(1) Cu^{+2} is more stable in water than Cu^+ due to more hydration energy

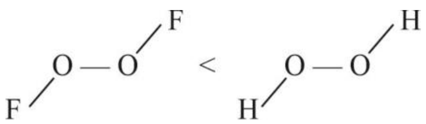
H.E. $\alpha \frac{e}{r}$

Hence, $(H.E.)_{Cu^{+2}} > (H.E.)_{Cu^+}$

4.(1) $K_2S_2O_8 + 2D_2O(l) \longrightarrow 2KDSO_4(aq). + D_2O_2(l)$

This method is used for the laboratory preparation of D_2O_2.

5.(4)



As electronegativity of $F > O$,

So, % S -character is less in O – F bond as compared to O – H bond,

So O – F bond is longer than O – H bond and similarly O – O bond is longer in H_2O_2.

Hence, $X \rightarrow$ longer and $Y -$ Shorter

6.(3) $K^+, Cl^-, Ca^{2+}, Sc^{3+}$ (No. of electrons = 18)

Isoelectronic species have the same number of electrons.

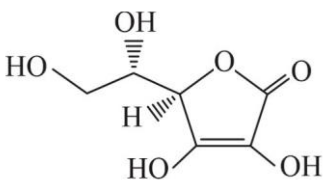
7.(1) Proper condition not given, incomplete question.

8.(4) Order of bond enthalpy (Kj/mol) is

$$\underset{348}{C-C} > \underset{297}{Si-Si} > \underset{260}{Ge-Ge} > \underset{240}{Sn-Sn}$$

9.(1) Industrial production of urea

10.(3) Most stable structure of vitamin C



11.(2) KOH obsorbs CO_2 gas when excess CO_2 is obsorbed by KOH, $KHCO_3$ is formed which remains soluble in the solution

12.(3) Oxygen is not present in Nessler's reagent (K_2HgI_4)

13.(2)

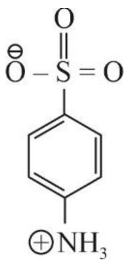
$$\log\frac{x}{m} = \frac{1}{n}\log P + \log k \rightarrow \text{Freundrich adsorption isotherm equations}$$

$$y = 3x + 2.505$$

On comparing both equation.

$$\frac{1}{n} = \text{slope} = 3,\ \log k = \text{intercept} = 2.505.$$

14.(1) Sulphanilic acid does not give Esterification test for Carboxyl group.



Sulphanilic acid contains N,S and C which gives NaSCN with $FeCl_3$ solution it gives blood red colouration because of $Fe(SCN)_3$ Complex

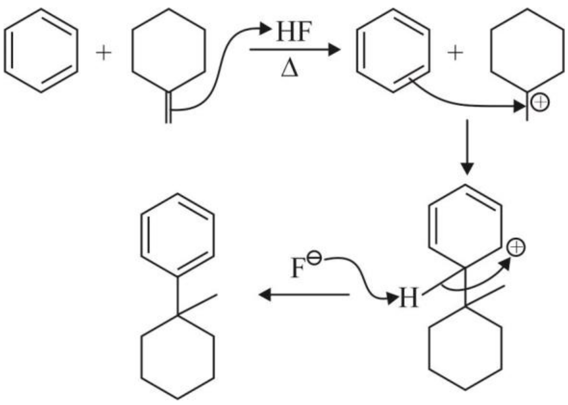
15.(2) Gypsum is used for making Fireproof wall boards

Gypsum is unstable at high temperatures

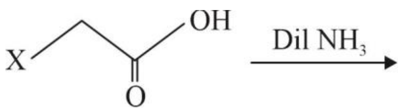
16.(4) $[Co(NH_3)_5NO_2]^{2+}$ shows linkage isomerism as $-NO_2$ is an ambidentate ligand.

$[Co(NH_3)_5NO_2]^{2+}$ and $[Co(NH_3)_5(ONO)]^{2+}$ two isomers.

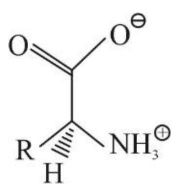
17.(1)



18.(3)



Amino acids exist in Zwitter ion form in Aqueous medium as



19.(1) $H_5C_6-\underset{C_6H_5}{\overset{C_6H_5}{C}}-Cl \xrightarrow[\text{Pyridine}]{[\bar{O}H]} H_5C_6-\underset{C_6H_5}{\overset{C_6H_5}{C^{\oplus}}} \xrightarrow{[\bar{O}H]} H_5C_6-\underset{C_6H_5}{\overset{C_6H_5}{C}}-OH$

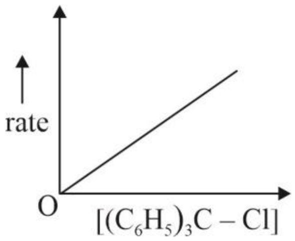
Tertiary Carbocation

Follow $S_N{}^1$ pathway

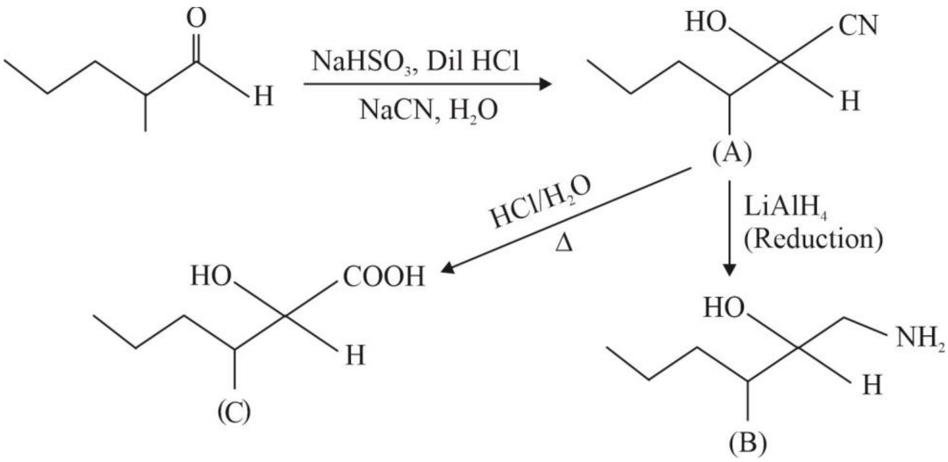
for $S_N{}^1$ reaction

Rate = K[substrate]

$= K\ [(C_6H_5)_3C-Cl]$



20.(1)



SECTION – 2

21.(6) $\left[Mn(H_2O)_6\right]^{2+}$

$Mn^{2+} = [Ar]3d^5$

Spin only magnetic moment $(\mu_{\text{spin only}}) = \sqrt{n(n+2)}\text{ B.M} = \sqrt{5(5+2)}\text{ B.M} = \sqrt{35}\text{ B.M} = 5.91\text{ B.M}$

22.(3) Copper matte is made up of a combination of copper sulfide (Cu_2S) and iron sulfide (FeS)

23.(4) Density $(d) = \dfrac{Zm}{N_a\ a^3}$ $\quad Z = 2(\text{Bcc})$ $\quad Z = 4(\text{Fcc})$

$$\frac{(d)_{Fcc}}{(d)_{Bcc}} = \frac{\dfrac{4\times m}{Na\,a^3_{Fcc}}}{\dfrac{2\times m}{Na\,a^3_{Bcc}}} = \frac{4\times m}{N_a\times 2^3}\times\frac{N_a\times(2.5)^3}{m\times 2} = \frac{15.625}{4} = 3.9$$

24.(372)

$\alpha = 0.20 \quad CH_3COOH \rightleftharpoons H^+ + CH_3COO^-$

for dissciation

$i = 1 + 0.20 \qquad\qquad i = 1 + \alpha$

molality (m) $= \dfrac{5 \times 1000}{60 \times 500} = \dfrac{1}{6}$

depression in freezing point:

$$\Delta T_F = ik_F m = 1.2 \times 1.86 \times \frac{1}{6} = 0.372 = 372 \times 10^{-3}\ ^\circ C$$

25.(139)

10% (V/V) means

Volume of solute $(Br_2) = 10$ ml

Volume of solution = 100 ml

Volume of solvent $(CCl_4) = 90$ ml

mass of $Br_2 = d \times V \Rightarrow 3.2 \times 10 \Rightarrow 32\,g$

Number of moles $= \dfrac{32}{160} = 0.2\,\text{moles}$

Molality $= \dfrac{\text{moles of solute}}{\text{mass of solvent (kg)}}$

$$\Rightarrow \frac{0.2 \text{ moles}}{1.6\,g\,cm^{-3} \times 90\,ml}$$

$$\Rightarrow \frac{0.2}{1.6 \times 90}\,mol/g \Rightarrow 10^3 \times \frac{0.2}{1.6 \times 90}\,mol/g \Rightarrow \frac{2 \times 10^2}{16 \times 9}$$

$$= 1.39\,mol/kg \Rightarrow 139 \times 10^{-2}\,mol/kg$$

26.(1006)

Given 0.3 g ethane (C_2H_6)

change in temperature $(\Delta T) = 0.5$

$$C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$$

$$\Delta ng = 2 - \left(1 + \frac{7}{2}\right)$$

$$\Rightarrow 2 - \frac{9}{2}$$

$$\Rightarrow -\frac{5}{2} \Rightarrow -2.5$$

$$q_v = c \times q \times M / m$$

$$\Rightarrow 20KJ\ K^{-1} \times 0.5K \times \frac{30g\,mol^{-1}}{0.3g}$$

$$\Rightarrow 20 \times 0.5 \times \frac{30}{0.3}\ KJ\ mol^{-1}$$

$$\Rightarrow 1000\,KJ\ mol^{-1}$$

$$q_p = q_v + \Delta ngRT$$

$$\Rightarrow 1000KJ\ mol^{-1} + 2.5 \times 8.314\ JK^{-1}\ mol^{-1} \times 298\,K$$

$$\Rightarrow 1000KJ\ mol^{-1} + 6193.93\ J\ mol^{-1} \Rightarrow 1000KJ\ mol^{-1} + 6193.93 \times 10^{-3} KJ\ mol^{-1}$$

$$\Rightarrow 1000KJ\ mol^{-1} + 6.1939KJ\ mol^{-1} \quad \Rightarrow 1006KJ\ mol^{-1}$$

27.(13039)

$$\kappa = \kappa_{Ag^+} + \kappa_{Br^-} + \kappa_{NO_3^-}$$

$$\kappa_{sp}\ \text{of AgBr} = \left[Ag^+\right]\left[Br^-\right]$$

$$4.9 \times 10^{-13} = \left[s + 10^{-5}\right][s]$$

$$\frac{4.9 \times 10^{-13}}{10^{-5}} = [s]$$

$$4.9 \times 10^{-8} = [s]$$

$$\left[Ag^+\right] = 4.9 \times 10^{-8} + 10^{-5} \approx 10^{-5} \text{mole / litre} = 10^{-2} \text{mole m}^{-3}$$

$$\left[Br^-\right] = 4.9 \times 10^{-8} \text{mole / litre} = 4.9 \times 10^{-5} \text{mole m}^{-3}$$

$$\left[NO_3^-\right] = 10^{-5} \text{mole / litre} = 10^{-2} \text{mole m}^{-3}$$

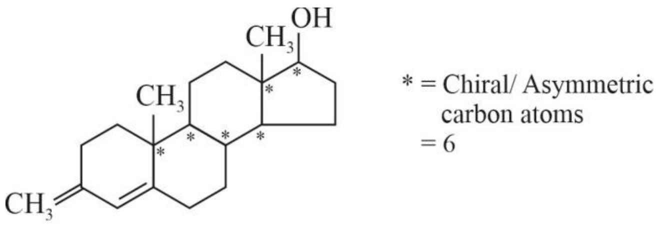
$$\kappa_{Ag^+} = 10^{-2} \times 6 \times 10^{-3} = 6 \times 10^{-5} = 6000 \times 10^{-8}$$

$$\kappa_{Br^-} = 4.9 \times 8 \times 10^{-3} \times 10^{-5} = 39.2 \times 10^{-8} = 39.2 \times 10^{-8}$$

$$\kappa_{NO_3^-} = 7 \times 10^{-3} \times 10^{-2} = 7 \times 10^{-5} = 7000 \times 10^{-8}$$

$$\kappa = 6000 \times 10^{-8} + 39.2 \times 10^{-8} + 7000 \times 10^{-8} \ = 10^{-8} \times 13039.2\ sm^{-1}$$

28.(6) Asymmetric carbon atoms has four different groups attached.



Testisteribe

29.(75) $A \longrightarrow B$ (zero order reaction)

$t_{1/2}\,(\text{half life}) = 50\,\text{min}$

For zero order $[A]_0 - [A]_t = Kt$ ...(1)

$\because$ $[A]_0 =$ initial conc.

$[A]_t =$ conc. at time 't'

$$t_{1/2} = \frac{[A]_0}{2k} \quad ...(2)$$

Also, Time required to reduce $\frac{1}{4}$ th of $[A]_0$

$$[A]_0 - \frac{[A]_0}{4} = k\ t_{1/4}$$

$$t_{1/4} = \frac{3[A]_0}{4K} \quad ...(3)$$

dividing equation (3) by (2), get

$$\frac{t_{1/4}}{t_{1/2}} = \frac{\frac{2}{4}\frac{[A]_0}{k}}{\frac{[A]_0}{2k}} = \frac{3}{2}$$

$$t_{1/4} = \frac{3}{2} \times t_{1/2} = \frac{3}{2} \times 50 = 75\ \text{min}$$

30.(3) Chloroliazepoxide, veronal , valium are Tranquilizers.

MATHEMATICS

SECTION - 1

1.(3) Let equation of required plane be

$$(2x+3y-z-2)+\lambda(x+2y+3z-6)=0$$

$$\Rightarrow (2+\lambda)x+(3+2\lambda)y+(-1+3\lambda)z-2-6\lambda=0 \qquad ...(1)$$

since (1) is perpendicular to $2x+y-z+1=0$

$$2(2+\lambda)+1(3+2\lambda)-1(-1+3\lambda)=0$$

$$8+\lambda=0 \Rightarrow \lambda=-8$$

$\therefore$ required plane is $-6x-13y-25z+46=0$

$$d=\frac{|(-6)(-7)-13(1)-25(1)+46|}{\sqrt{(-6)^2+(-13)^2+(-25)^2}}=\frac{50}{\sqrt{830}}$$

$$d^2=\frac{2500}{830}=\frac{250}{83}$$

2.(4) **(A)**

p	q	$p \wedge q$	$p \vee (p \wedge q)$ $\rightarrow$ (*not a tautology*)
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

(B)

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$p \rightarrow (p \wedge (p \rightarrow q)$ $\rightarrow$ (*not a tautology*)
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

(C)

p	q	$(p \wedge (p \rightarrow q))$	$\sim q$	$(p \wedge (p \rightarrow q)) \rightarrow \sim q$ (*not a tautology*)
T	T	T	F	F
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T

(D)

p	q	$p \wedge q$	$\sim p \rightarrow q$	$(p \wedge q) \rightarrow (\sim p \rightarrow q)$ (*tautology*)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

2nd Method

$p \wedge q \rightarrow (\sim p \rightarrow q)$

Apply $P \rightarrow Q = \sim P \vee Q$

$(p \wedge q) \rightarrow (p \vee q)$

$= \sim (p \wedge q) \vee (p \vee q)$ (Tautology)

$\{(P \cap Q)' \cup (P \cup Q) = \cup\}$

3.(2) Let $I = \int_{-\pi/4}^{\pi/4} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$

$$I = \int_{-\pi/4}^{\pi/4} \frac{-x + \frac{\pi}{4}}{2 - \cos 2x} dx \quad \left[Using \int_a^b f(x) dx = \int_a^b f(a + b - x) \right]$$

$$2I = 2 \times \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{dx}{2 - \cos 2x}$$

$$I = \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{dx}{2 - (2\cos^2 x - 1)} = \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{dx}{3 - 2\cos^2 x}$$

$$= \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{\sec^2 x dx}{3\sec^2 x - 2} = \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{\sec^2 x dx}{3(1 + \tan^2 x) - 2}$$

$$= \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{\sec^2 x dx}{3\tan^2 x + 1} = \frac{\pi}{4\sqrt{3}} \tan^{-1}\left(\sqrt{3} \tan x\right)\Big|_{-\pi/4}^{\pi/4}$$

$$= \frac{\pi}{4\sqrt{3}} \left[\tan^{-1} \sqrt{3} - \tan^{-1}\left(-\sqrt{3}\right) \right]$$

$$= \frac{\pi}{4\sqrt{3}} \left[\frac{\pi}{3} - \left(\frac{-\pi}{3} \right) \right] = \frac{\pi}{4\sqrt{3}} \times \frac{2\pi}{3} = \frac{2\pi^2}{12\sqrt{3}} = \frac{\pi^2}{6\sqrt{3}}$$

4.(2) Total sample space $= n(S) = 6^2 = 36$

$A = \{(1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6)\}$

$B = \{(2,1), (2,3), (2,5), (4,1), (4,3)(4,5), (6,1), (6,3), (6,5)\}$

$C = \{(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)\}$

Now $n(A \cup B \cap C) = \{(1,2), (1,4), (1,6), (3,4), (3,6), (5,6)\} = 6$

5.(1) $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow (\vec{r} \times \vec{a}) - (\vec{c} \times \vec{a}) = \vec{0}$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = \vec{0}$

$\Rightarrow (\vec{r} - \vec{c}) = \lambda \vec{a}$

$\vec{r}=\vec{c}+\lambda\vec{a}$ ...(1)

Given, $\vec{r}.\vec{b}=0$

from (1) $\vec{r}.\vec{b}=\vec{c}.\vec{b}+\lambda\vec{a}.\vec{b}$

$\Rightarrow(1-3)+\lambda(2+5)=0$

$\Rightarrow 7\lambda=2\Rightarrow\lambda=\frac{2}{7}$

$\vec{r}=(\hat{i}+2\hat{j}-3\hat{k})+\frac{2}{7}(2\hat{i}+7\hat{j}+5\hat{k})$

$\Rightarrow\frac{(7\hat{i}+14\hat{j}-21\hat{k})+4\hat{i}-14\hat{j}+10\hat{k}}{7}$

$\vec{r}=\frac{11\hat{i}-11\hat{k}}{7}$

$|\vec{r}|=\sqrt{\frac{121}{49}+\frac{121}{49}}=\sqrt{2\times\frac{121}{49}}\Rightarrow\frac{11}{7}\sqrt{2}$

6.(2) $R_1:(A\cap B')\cup(B\cap A')=\phi$

$\Rightarrow A=B$

$R_2:(A\cup B')=(B\cup A')$

$\Rightarrow A=B$

Hence, R_1 *and* R_2 are equivalence.

7.(3) $A=\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}=A=\begin{pmatrix}\cos\frac{\pi}{3} & \sin\frac{\pi}{3}\\ -\sin\frac{\pi}{3} & \cos\frac{\pi}{3}\end{pmatrix}$

Let $A=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$

then $A^n=\begin{pmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{pmatrix}$ $n\in N$

$A^{30}=\begin{pmatrix}\cos 10\pi & \sin 10\pi\\ -\sin 10\pi & \cos 10\pi\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I$

$A^{25}=\begin{pmatrix}\cos\frac{25\pi}{3} & \sin\frac{25\pi}{3}\\ -\sin\frac{25\pi}{3} & \cos\frac{25\pi}{3}\end{pmatrix}$

Note: $a=\cos\left(\frac{25\pi}{3}\right)$

$$\Rightarrow \cos\left(8\pi+\frac{\pi}{3}\right)=\cos\frac{\pi}{3}$$

$$b=\sin\left(\frac{25\pi}{3}\right)=\sin\left(8\pi+\frac{\pi}{3}\right)=\sin\frac{\pi}{3}$$

$$A^{25}=\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}=A$$

$$A^{30}=I \quad ...(1)$$

$$A^{25}=A \quad ....(2)$$

Add (1) and (2)

$$A^{30}+A^{25}=A+I\Rightarrow A^{30}+A^{25}-A=I$$

8.(4)

$$\frac{dy}{dx}=\frac{1-xy^2}{2x^2y}=\frac{1}{2x^2y}-\frac{xy^2}{2x^2y}$$

$$\frac{dy}{dx}=\frac{1}{2x^2y}-\frac{y}{2x}$$

$$\frac{dy}{dx}+\frac{y}{2x}=\frac{1}{2x^2y}$$

$$y\frac{dy}{dx}+\frac{y^2}{2x}=\frac{1}{2x^2}$$

$$Let\ \boxed{y^2=z} \qquad 2y\frac{dy}{dx}=\frac{dz}{dx}$$

$$\frac{1}{2}\frac{dz}{dx}+\frac{z}{2x}=\frac{1}{2x^2}$$

$$I.F=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$$

$$z.x=\int\frac{1}{x^2}.x\ dx$$

$$z.x=\ln x+\ln c$$

$$xy^2=\ln x+\ln c$$

Given at $x=2, y=\sqrt{\ln 2}$

$$2\ln 2=\ln 2+\ln C$$

$$\ln 4-\ln 2=\ln C$$

$$\ln 2=\ln C$$

$$C=2$$

$$xy^2=\ln x+\ln 2=xy^2=\ln(2x)=2x=e^{xy^2}$$

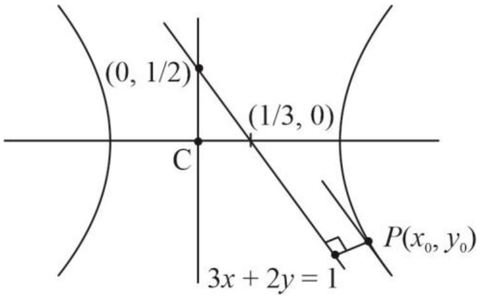
By comparing

$\alpha = 2;\ \beta = 1;\ \gamma = 2$

Then $\alpha + \beta - \gamma$

$3 - 2 = 1$

9.(2)



Shortest distance lies along common normal, differentiating $3x^2 - 4y^2 = 36$ w.r.t. x, and put

$x = x_0;\ \ y = y_0$

$$\frac{dy}{dx} = \frac{3x_0}{4y_0} = -\frac{3}{2}$$

$\Rightarrow\ x_0 = -2y_0$ ...(1)

(x_0, y_0) also lies on the curve $3x^2 - 4y^2 = 36$

$= 3x_0^2 - 4y_0^2 = 36$ ...(2)

$= 3 \times 4y_0^2 - 4y_0^2 = 36$

$= 8y_0^2 = 36$

$= y_0^2 = \frac{9}{2} \Rightarrow y_0 = \pm\frac{3}{\sqrt{2}}$

put $y_0^2 = \frac{9}{2}$ in equation (2)

$3x_0^2 - 4y_0^2 = 36$

$\Rightarrow x_0 = \mp 3\sqrt{2}$

Req. point lies in 4th quadrant:

$x_0 = 3\sqrt{2},\ \&\ y_0 = -\frac{3}{\sqrt{2}}$

Now $\sqrt{2}(y_0 - x_0) = \sqrt{2}\left(-\frac{3}{\sqrt{2}} - 3\sqrt{2}\right) = -3 - 6 = -9$

10.(2) Put $x = \tan\theta \Rightarrow$ given $0 < x < 1\ \&\ \theta = \tan^{-1}x \Rightarrow 0 < \theta < \frac{\pi}{4} \Rightarrow \theta \in \left(0, \frac{\pi}{4}\right)$

$$2\tan^{-1}\left(\frac{1 - \tan\theta}{1 + \tan\theta}\right) = \cos^{-1}\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right)$$

$\Rightarrow 2\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)=\cos^{-1}(\cos 2\theta)$

$2\left(\frac{\pi}{4}-\theta\right)=2\theta$

$\Rightarrow \frac{\pi}{2}-2\theta=2\theta$

$\Rightarrow 4\theta=\frac{\pi}{2}\Rightarrow \theta=\frac{\pi}{8}$

$\tan\frac{\pi}{8}=\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}=\sqrt{2}-1$

$\alpha x+y+z=1$

11.(1) $x+\alpha y+z=1$

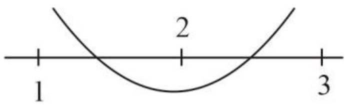
$x+y+\alpha z=\beta$

for infinite solution

$\alpha=1, \& \beta=1$

Note: For $\alpha=1 \& \beta=1$ given three planes overlap to each other and provide infinitely many solution.

12.(1)



$f(1)f(2)<0$

$\Rightarrow (k-6)(k-8)<0$

$k\in(6, 8)$ ....(1)

and $f(2)f(3)<0$

$\Rightarrow (k-8)(k-6)<0$

$6<k<8$

$=k\in(6,8)$ ...(2)

$(i)\cap(ii) k\in(6,8)$ $k=7$ is only possible integer

13.(3) $f(x)=x^x,\ x>0$

$\ln y=x\ln x$

$\frac{1}{y}.\frac{dy}{dx}=x\times\frac{1}{x}+\ln x.1$

$\frac{dy}{dx}=y(1+\ln x)$

$\Rightarrow y'(x)=x^x(1+\ln x)$

$\Rightarrow y'(2)=4(1+\ln 2)$

$2y'(2)=8(1+\ln 2)$ ...(1)

$y'(x)=y(1+\ln x)$

$y''(x) = y \times \frac{1}{x} + (1 + \ln x)\, y'$

$\Rightarrow \frac{x^x}{x} + (1 + \ln x)(x^x)(1 + \ln x)$

$y''(2) = 2 + 4(1 + \ln 2)^2$

$2y'(2) = 8(1 + \ln 2)$

$= 2 + 4(1 + \ln 2)[1 + \ln 2 - 2]$

$\Rightarrow 2 + 4(1 + \ln 2)(\ln_2 - 1)$

$\Rightarrow 2 + 4(\ln 2)^2 - 1)$

$\Rightarrow 2 + 4(\ln 2)^2 - 4$

$4(\ln 2)^2 - 2$

14.(4) Given $x_1 = 9$

mean $\bar{X} = \frac{x_1 + x_2 + ... + x_7}{7}$

$\Rightarrow \frac{a + a + d + ... + a + 6d}{7}$

$\Rightarrow \frac{7a + d(1 + 2 + ... + 6)}{7}$

$\Rightarrow \frac{7a + d \times \frac{6}{2}(7)}{7}$

$\frac{7a + 21d}{7}$

$\Rightarrow a + 3d = a_4 = \bar{x} \quad ...(1)$

$\sigma^2 = 16 = \sum \frac{(x_i - \bar{x})^2}{N} \quad ....(2)$

(i) Put $i = 1,\ x_1 - \bar{x}$

$\Rightarrow a - (a + 3d) = -3d$

(ii) Put $i = 2, x_2 - \bar{x} = (a + d) - (a + 3d) \Rightarrow -2d$

(iii) Put $i = 3,\ x_3 - \bar{x} = (a + 2d) - (a + 3d) \Rightarrow -d$

(iv) Put $i = 4,\ x_4 - \bar{x} = (a + 3d - (a + 3d) = 0$

(v) Put $i = 5,\ x_5 - \bar{x} = (a + 4d) - (a + 3d) \Rightarrow d$

(vi) Put $i = 6,\ x_6 - \bar{x} = (a + 5d) - (a + 3d) \Rightarrow 2d$

(vii) Put $i = 7,\ x_7 - \bar{x} = (a + 6d) - (a + 3d) \Rightarrow 3d$

Put in equation (2)

$$\frac{2\left[d^2+4d^2+9d^2\right]}{7}=16$$

$14d^2=8\times 7\Rightarrow d=2$ and $a=9$

$\vec{x}=9+3\times 2\Rightarrow 15$

$x_6=a+5d\Rightarrow 9+5\times 2=19$

$15+19=34$

15.(2) $f(x)+f\left(\frac{1}{1-x}\right)=1+x$

put $x=2$, $f(2)+f(-1)=3$

put $x=-1$, $f(-1)+f\left(\frac{1}{2}\right)=0$

$\quad -\quad\quad -\quad\quad -$

subtract $f(2)-f\left(\frac{1}{2}\right)=3$ ...(1)

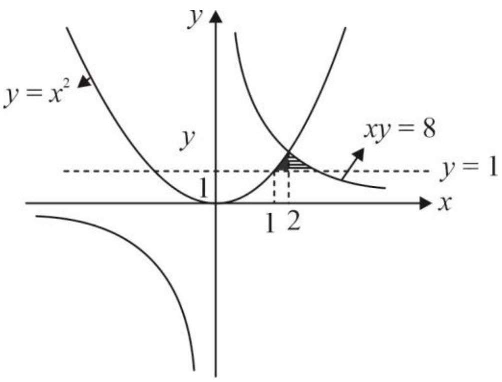
put $x=\frac{1}{2}$, $f\left(\frac{1}{2}\right)+f(2)=\frac{3}{2}$ ...(2)

add (1) and (2), $2f(2)=3+\frac{3}{2}$

$=2f(2)=\frac{9}{2}=f(2)=\frac{9}{4}$

16.(4)

$$\left.\begin{array}{l}xy=8\\ y=x^2\end{array}\right\}x=2, y=4$$



Required Area $=\int_1^4\left(\frac{8}{y}-\sqrt{y}\right)dy=8\ln y-\left.\frac{y^{3/2}}{\frac{3}{2}}\right|_1^4=\left(8\ln 4-\frac{2}{3}\times 4^{3/2}\right)-\left(0-\frac{2}{3}\right)$

$=8\ln 4-\frac{2}{3}\times 8+\frac{2}{3}=16\ln 2-\frac{14}{3}$

17.(4) Projection of $\vec{a}$ on $\vec{b}$ is $\vec{a}.\hat{b}$

$$=\left(5\hat{i}-\hat{j}-3\hat{k}\right).\frac{(\hat{i}+3\hat{j}+5\hat{k})}{\sqrt{35}}$$

$$=\frac{5-3-15}{\sqrt{35}}=\frac{-13}{\sqrt{35}}$$

Since proejction is -ve,

$\therefore$ projection of $\vec{a}$ on $\vec{b}$ is in direction opposite to that of $\vec{b}$.

As no option matches, therefore it should be Bonus.

18.(3) $$\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}=\frac{1}{2}\sum_{n=1}^{\infty}\frac{2n(2n-1)+8n+8}{(2n)!}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{2n(2n-1)}{(2n)!}+\frac{8n}{(2n)!}+\frac{8}{(2n)!}\right]$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{1}{(2n-2)!}+\frac{4}{(2n-1)!}+\frac{8}{(2n)!}\right]$$

$$\frac{1}{2}\left(1\left[1+\frac{1}{2!}+\frac{1}{4!}+...\right]+4\left[\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+...\right]+8\left[\frac{1}{2!}+\frac{1}{4!}+...\right]\right) \quad ...(1)$$

we know $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+.....$

$$e^1=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....$$

$$e^{-1}=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....$$

$$\frac{e+e^{-1}}{2}=1+\frac{1}{2!}+\frac{1}{4!}+....$$

$$\frac{e-e^{-1}}{2}=\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+....$$

putting in (1)

$$\frac{1}{2}\left(1\left(\frac{e+e^{-1}}{2}\right)+4\left(\frac{e-e^{-1}}{2}\right)+8\left(\frac{e+e^{-1}}{2}-1\right)\right)$$

$$\frac{13e}{4}+\frac{5}{4e}-4$$

19.(1) $$f(x)=\begin{cases}-\left(x^2-5x+6\right)-3x+2 & 2\le x\le 3\\ x^2-5x+6-3x+2 & -1\le x<2\end{cases}$$

$$f(x)=\begin{cases}-x^2+2x-4 & 2\le x\le 3\\ x^2-8x+8 & -1\le x<2\end{cases}$$

$f(x)=-x^2+2x-4 \quad in \quad x\in[2,3]$

$f'(x)=-2x+2<0 \quad for \quad x\in[2,3]$

$f(x)=x^2-8x+8 \quad in \quad x\in[-1,2)$

$f'(x)=2x-8<0 \quad for \quad x\in[-1,2)$

$\therefore\ f(-1)=17$ absolute maximum value

$f(3)=-7$ absolute minimum value

sum $=17+(-7)=10$

20.(2) $\left|\frac{1+ai}{b+i}\right|=1\Rightarrow|1+ai|^2=|b+i|^2$

$(1+ai)(1-ai)=(b+i)(b-i)$

$1+a^2=b^2+1\Rightarrow a=\pm b$

Since $ab<0, \therefore a=-b$ ....(1)

Using

$|z-1|=|2z|$

$\Rightarrow|z-1|^2=(2|z|)^2$

$\Rightarrow(z-1)(\bar{z}-1)=4z\bar{z}$

$\Rightarrow z\bar{z}-z-\bar{z}+1=4z\bar{z}$

$3z\bar{z}+z+\bar{z}-1=0$

$3|z|^2+z+\bar{z}-1=0$ ....(2)

Since $z=a+ib$ satisfies (2)

$3(a^2+b^2)+(a+ib)+(a-ib)-1=0$

$3(a^2+b^2)+2a-1=0$ ...(3)

Solving (1) and (3)

we get $3(a^2+b^2)+2a-1=0$

$3(a^2+b^2)+2a-1=0$

$6a^2+2a-1=0$

$a=\frac{-2\pm\sqrt{28}}{12}=\frac{-2\pm2\sqrt{7}}{12}$

Case-I

$a=\frac{-1+\sqrt{7}}{6}$

$[a]=0$

$b=-a=\frac{1-\sqrt{7}}{6}$

$\therefore\ \frac{1+[a]}{4b}=\frac{1+0}{4\left(\frac{1-\sqrt{7}}{6}\right)}=\frac{3}{2(1-\sqrt{7})}$

Case-2

$a=\frac{-1-\sqrt{7}}{6}$

$[a]=-1$

$b=-a=\frac{1+\sqrt{7}}{6}$

$\therefore\ \frac{1+[a]}{4b}=\frac{1-1}{4\left(\frac{1+\sqrt{7}}{6}\right)}=0$

As no option matches, therefore it should be Bonus.

SECTION – 2

21.(105)

$x \geq 1,\ y \geq 3,\ z \geq 4$

$x-1 \geq 0,\ y-3 \geq 0,\ z-4 \geq 0$

$Let\ \ x-1 = X$

$y-3 = Y$

$z-4 = Z$

$\therefore X \geq 0,\ Y \geq 0,\ Z \geq 0$

$x+y+z = 21$

$(X+1)+(Y+3)+(Z+4) = 21$

$X+Y+Z = 13$

Not of integral solution $= {}^{13+3-1}C_{3-1} = {}^{15}C_2 = 105$

22.(16) $y^2 - 4y = 8x + 4$

$(y-2)^2 = 8(x+1)$

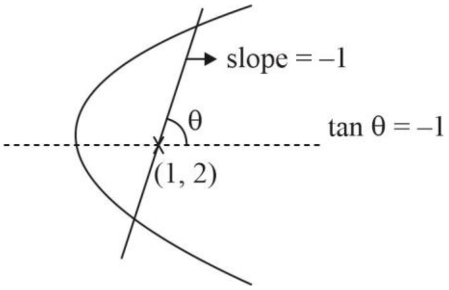
$a = 2,$ vertex (–1, 2), focus (1, 2)

Since x-int of focal chord is 3

$\therefore$ It passes through (3, 0)

equation of focal chord is $y - 0 = -1(x-3)$

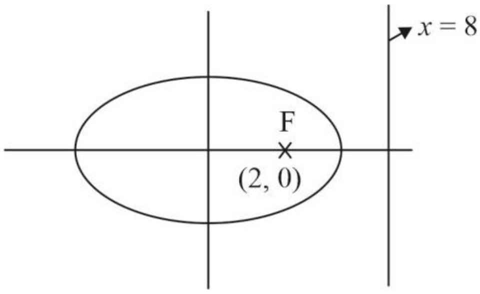
$x + y = 3$



Length of focal chord = $4a\ cosec^2\theta$

$= 4 \times 2 \times \left[1 + (-1)^2\right] = 16$

23.(39)



$$\left.\begin{array}{l} ae=2 \\ \dfrac{a}{e}=8 \end{array}\right\} a=4,\ e=1/2$$

$$a^2e^2=a^2-b^2$$

$$\Rightarrow 4=16-b^2$$

$$b^2=12$$

$$b=2\sqrt{3}$$

$$E:\frac{x^2}{16}+\frac{y^2}{12}=1$$

Let $P(4\cos\theta,\ 2\sqrt{3}\sin\theta)$ be a point in 1st quadrant equation of tangent is

$$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{2\sqrt{3}}=1$$

It passes through $(0,\ 4\sqrt{3})$

$$2\sin\theta=1\Rightarrow \sin\theta=\frac{1}{2}$$

$$\therefore \cos\theta=\frac{\sqrt{3}}{2}$$

$$P(2\sqrt{3},\ \sqrt{3})$$

for Q put $y=0$

$$x=4\sec\theta=\frac{8}{\sqrt{3}}$$

$$Q\left(\frac{8}{\sqrt{3}},0\right)$$

$$PQ=\sqrt{\left(2\sqrt{3}-\frac{9}{\sqrt{3}}\right)^2+\left(\sqrt{3}-0\right)^2}=\sqrt{\frac{4}{3}+3}=\sqrt{\frac{13}{3}}$$

$$(3PQ)^2=9\times\frac{13}{3}=39$$

24.(4) $\sqrt{2^{\log_2(10-3^x)}}=\left(10-3^x\right)^{1/2}$

$$\sqrt[5]{2^{(x-2)\log_2 3}}=\left(3^{(x-2)}\right)^{1/5}=3^{\frac{x-2}{5}}$$

$$T_6={}^mC_5\left(10-3^x\right)^{\frac{m-5}{2}}\left(3^{\frac{x-2}{5}}\right)^5={}^mC_5\left(10-3^x\right)^{\frac{m-5}{2}}.3^{x-2}=21 \qquad ...(1)$$

${}^mC_1,\ {}^mC_2,\ {}^mC_3$ are 1st, 3rd, 5th terms of A.P.

$$a={}^mC_1$$

$a+2d={}^{m}C_2$

$a+4d={}^{m}C_3$

$\therefore \quad {}^{m}C_3-{}^{m}C_2={}^{m}C_2-{}^{m}C_1$

${}^{m}C_3+{}^{m}C_1=2\,{}^{m}C_2$

$$\frac{m(m-1)(m-2)}{6}+m=\frac{2m(m-1)}{2}$$

$$\frac{(m-1)(m-2)}{6}+1=m-1$$

$m^2-3m+2=6(m-2)$

$m^2=9m+14=0 \Rightarrow m=7,\ 2\ (rejected)$

$\therefore\ m=7,\ put\ \ m=7\ in\ (1)$

${}^{7}C_5\left(10-3^x\right)^1 3^{x-2}=21$

$$\left(10-3^x\right).\frac{3^x}{9}=1$$

$(10-t)t=9$

$t^2-10t+9=0$

$t=1,\ 9$

$3^x=1,\ 9 \Rightarrow x=0,\ 2$

Required sum $=0^2+2^2=4$

25.(6) $\alpha x+\beta y+yz=1$ is not an equation of plane.

Correct equation of plane should be $\alpha x+\beta y+\gamma z=1$.

There is printing mistake in question. Instead of *yz*, it should be γz .

26.(1) $T_{r+1}={}^{22}C_r\left(x^{2/3}\right)^{22-r}.\left(\frac{\alpha}{x^3}\right)^r$

Exponent of $x=\frac{2}{3}(22-r)-3r=0$

$r=4$

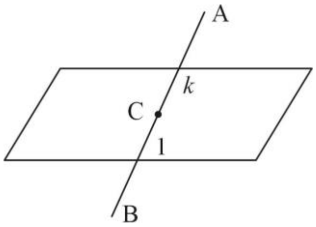
$\therefore T_5={}^{22}C_4\times\alpha^4$

$$7315=\frac{22.21.20.19}{4!}\alpha^4$$

$1=\alpha^4$

$\therefore |\alpha|=1$

27.(10) Equation of AB $\dfrac{x+3}{5}=\dfrac{y+6}{10}=\dfrac{z-1}{-4}$



Let Any pt of on AB be $(5\lambda-3, 10\lambda-6, -4\lambda+1)$ ...(1)

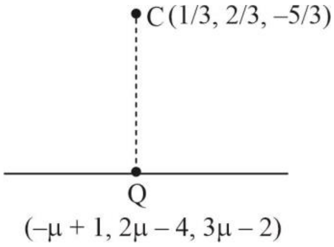
$9(5\lambda-3)+(10\lambda-6)+2(-4\lambda+1)=0$

$42\lambda-28-0$

$\lambda=\dfrac{2}{3}$

put $\lambda=\dfrac{2}{3}$ in (1), $C\left(\dfrac{1}{3},\dfrac{2}{3},\dfrac{-5}{3}\right)$

Now any pt. on line 2 $\dfrac{x-1}{-1}=\dfrac{y+4}{2}=\dfrac{z+2}{3}$



$bc\ Q(-\mu+1, 2\mu-4, 3\mu-2)$

$\overrightarrow{CQ}.\vec{b}=0$

$\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$

$\left(-\mu+1-\dfrac{1}{3}\right).(-1)+\left(2\mu-4-\dfrac{2}{3}\right).(2)+\left(3\mu-2+\dfrac{5}{3}\right).3=0$

$\mu-\dfrac{2}{3}+4\mu-\dfrac{28}{3}+9\mu-1=0$

$14\mu=11\Rightarrow\mu=\dfrac{11}{14}$

d.r's of foot of $\perp\left(-\mu+\dfrac{2}{3}, 2\mu-\dfrac{14}{3}, 3\mu-\dfrac{1}{3}\right)$

$\left(\dfrac{-11}{14}+\dfrac{2}{3},\dfrac{22}{14}-\dfrac{14}{3},\dfrac{3\times11}{14}-\dfrac{1}{3}\right)$

$\left(\dfrac{-5}{42},\dfrac{-130}{42},\dfrac{85}{42}\right)$ or $(-1, -26, 17)$

$a=-1, b=-26, c=17$

$|a+b+c|=10$

28.(81)

$------4$ fixed

Case-1 $'4' \rightarrow 6$ times Number $=1$

Case-2 $'4' \rightarrow 4$ times

$'5' \rightarrow 1$ times

$'9' \rightarrow 1$ times

Numbers $= \frac{5!}{3!} = 20$

Case 3 $'4' \rightarrow 2$ times

$'5' \rightarrow 2$ times

$'9' \rightarrow 2$ times

Number $= \frac{5!}{2!2!} = 30$

Case 4 $'4' \rightarrow 1$ time

$'5' \rightarrow 1$ time

$'9' \rightarrow 4$ time

Numbers $= \frac{5!}{4!} = 5$

Case 5 $'4' \rightarrow 3$ times

$'9' \rightarrow 3$ times

Numbers $= \frac{5!}{3!2!} = 10$

Case 6 $'4' \rightarrow 1$ time

$'5' \rightarrow 4$ times

$'9' \rightarrow 1$ time

Numbers $= \frac{5!}{4!} = 5$

Case 7 $'4' \rightarrow 3$ times

$'5' \rightarrow 3$ times

Numbers $= \frac{5!}{3!2!} = 10$

Total number 81

29.(13) $I=\int_0^{\pi}\frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)}{1+5^{\cos x}}dx$

$$I=\int_0^{\pi/2}\left(\frac{5^{\cos x}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x}{1+5^{\cos x}}+\frac{5^{\cos(\pi-x)}(1+\cos(\pi-x)\cos(3(\pi-x))+\cos^2(\pi-x)+\cos^3(\pi-x)\cos 3(\pi-x)}{1+5^{\cos(\pi-x)}}\right)dx$$

$$I=\int_0^{\pi/2}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx$$

$$=\int_0^{\pi/2}\left(1+\cos x\left(4\cos^3 x-3\cos x\right)+\cos^2 x+\cos^3 x(4\cos^3 x-3\cos x)\right)dx$$

$$=\int_0^{\pi/2}\left(4\cos^6 x+\cos^4 x-2\cos^2 x+1\right)dx$$

$$=4\times\left(\frac{5\times3\times1}{6\times4\times2}\times\frac{\pi}{2}\right)+\left(\frac{3\times1}{4\times2}\times\frac{\pi}{2}\right)-2\left(\frac{\pi}{4}\right)+\frac{\pi}{2}=\frac{13\pi}{16}$$

$\therefore\ k=13$

30.(321)

3, 7, 11, 15, ----, 399, $d_1=4$

2, 5, 8, 11, ----, 359, $d_2=3$

2, 7, 12, 17,----, 197 $d_3=5$

Common terms of 3 A.P's will be in A.P.

whose common difference is L.C.M of {4, 3, 5} = 60

First common term is 47 (by observation)

common terms are 47, 107, 167

sum = 321