

SOLUTIONS OF JEE ADVANCED 2025 | PAPER - 2

PHYSICS

1.(B) $z = \frac{s^2 \sigma}{k}; \quad R = \frac{1}{\sigma} \times \frac{\ell}{A}; \quad \sigma = \frac{1}{mR}$

For $P = \frac{T}{\ell} \times Ak \Rightarrow \frac{P}{Tm}$

$$z = \frac{\frac{v^2}{T^2} \times \sigma}{k} = \frac{\frac{I^2 R^2}{T^2} \times \frac{1}{mR}}{P/Tm} = \frac{P}{T \times P} = \frac{1}{T} \quad = [M^0 L^0 T^0 I^0 K^{-1}]$$

2.(C) $\cos \theta_1 = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}; \quad \theta_1 = \frac{\pi}{4}$

$\cos \theta_2 = \frac{R}{2R} = \frac{1}{2}; \quad \theta_2 = \frac{\pi}{3}$

$Q + Q_i = Q - \frac{4Q}{5} = \frac{Q}{5}$ (net charge)

$$E = \left(\frac{\frac{Q}{5}}{2\pi\epsilon_0 r} \right)$$

$$d\phi = \left(\frac{Q}{10\pi\epsilon_0 r} \times \frac{rd\theta}{\cos\theta} \times \ell \right) \times \cos\theta$$

$$\int d\phi = \frac{\theta\ell}{10\pi\epsilon_0} \int_{\pi/4}^{\pi/3} d\theta = \frac{\theta\ell}{10\pi\epsilon_0} \left(\frac{\pi}{3} - \frac{\pi}{4} \right) = \frac{Q\ell}{10\pi\epsilon_0} \frac{\pi}{12}$$

Total flux will be twice of this $= \frac{Q\ell}{60\epsilon_0}$

Better way avoiding integration

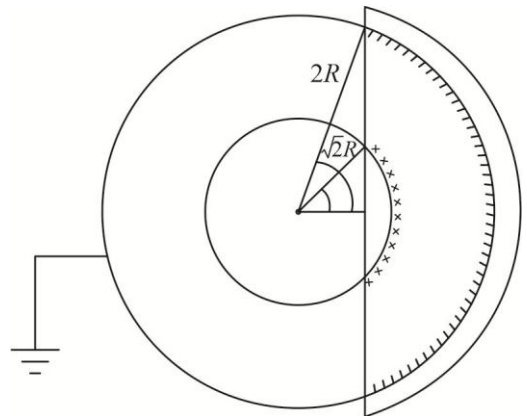
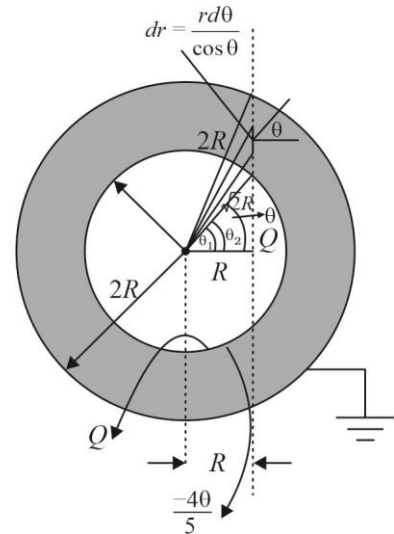
$$Q(-ve) = \frac{-Q \times 120}{360} = -\frac{Q}{3}$$

$$Q(+ve) = \frac{Q \times 90}{360} = \frac{Q}{4}$$

ϕ_{through} curved surface will be zero, as we can't have field outside

Assumed gaussian surface

$$\phi_{\text{curved}} + \phi_{\text{plane}} = \frac{(\theta/3 - \theta/4)}{5\epsilon_0}; \quad \phi_{\text{plane}} = \frac{Q}{60\epsilon_0}$$



3.(C) $\tau = kxL + \frac{kx}{2} \times \frac{L}{2}$

$$\tau = kxL \left(1 + \frac{1}{4} \right)$$

$$\tau = kxL \times \frac{5}{4}$$

$$\tau = kL^2 \times \frac{5}{4} \theta$$

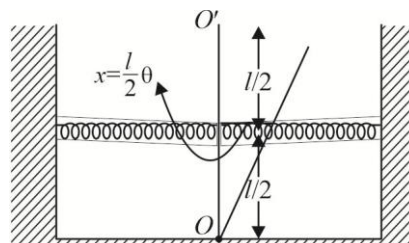
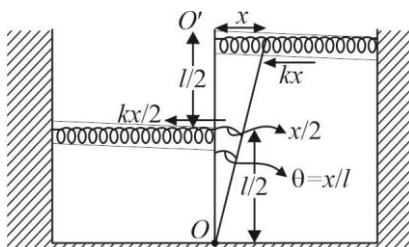
$$I \frac{d^2\theta}{dt^2} + kL^2 \times \frac{5}{4} \theta = 0$$

Both springs in parallel equivalent

Spring = $2k$ (effective)

$$\tau = 2k \frac{\ell}{2} \times \theta \times \frac{L}{2} = \frac{2kL}{2} \times \frac{x}{\ell} \times \frac{L}{2}$$

$$= \frac{kLx}{2} = \frac{kL^2}{2} \theta; \quad I \frac{d^2\theta}{dt^2} + \frac{kL^2}{2} \theta = 0$$



4.(B) $\frac{Gm_1m_2}{r^2} = m_2\omega^2 \frac{m_1r}{m_1+m_2}$

$$L = \mu r^2 \omega = \frac{m_1m_2}{m_1+m_2} r^2 \frac{\sqrt{G(m_1+m_2)}}{r\sqrt{r}}$$

$$L = \frac{m_1m_2}{\sqrt{m_1+m_2}} \sqrt{r} \sqrt{G}$$

As m_1+m_2 is constant and L is also constant so, $m_1m_2\sqrt{r}$ is constant

$$m_1m_2\sqrt{r} = c$$

$$\ln m_1 + \ln m_2 + \frac{1}{2} \ln r = C$$

$$\frac{1}{m_1} \frac{dm_1}{dt} + \frac{1}{m_2} \frac{dm_2}{dt} + \frac{1}{2r} \frac{dr}{dt} = 0$$

$$\frac{1}{m_1} (\gamma) + \frac{1}{m_2} (-\gamma) + \frac{1}{2r} \frac{dr}{dt} = 0$$

For $m_1 \gg m_2$

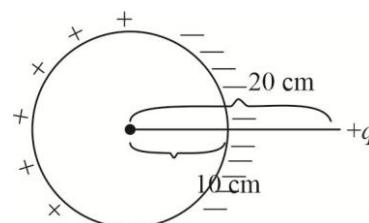
$$\frac{1}{r} \frac{dr}{dt} = \frac{2\gamma}{m_2}$$

5.(ABC) Potential at center = potential of conductor at any point

$$= \frac{kq}{20\text{cm}} = \frac{9 \times 10^9 \times 10^{-8}}{20 \times 10^{-2}} = 450V$$

Since conductor is earth, its potential must be zero.

$$\Rightarrow \frac{kQ}{R} + \frac{kq}{20} = \text{potential at center of sphere}$$



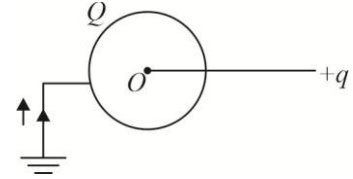
= potential at surface = 0

$$\Rightarrow Q = -\frac{q}{20} \times 10 \Rightarrow Q = -5 \times 10^{-9} C$$

After grounding is removed the charge on surface will remain

$$= -5 \times 10^{-9} C$$

$$\text{Final potential} = \frac{kx(-5) \times 10^{-6}}{10 \times 10^{-2}} + 450 = 0$$



6.(AB) Case -1

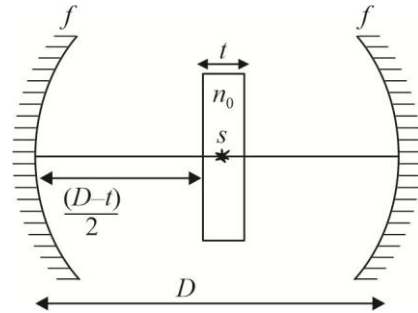
$$\frac{1}{\mu} \frac{t}{2} + \frac{(D-t)}{2} = 2f; \quad \frac{t}{2\mu} - \frac{t}{2} + \frac{D}{2} = 4f$$

$$\Rightarrow D = 4F + t \left(1 - \frac{1}{\mu} \right)$$

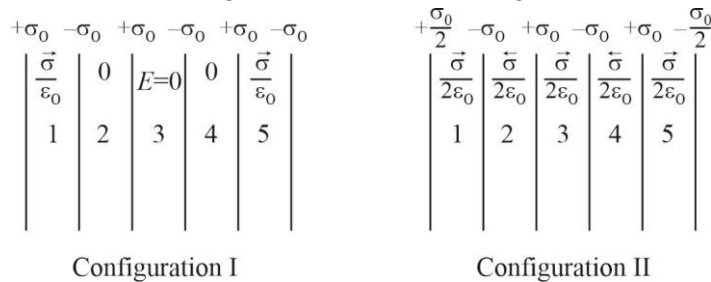
Case -2

$$\frac{(D-t)}{2} + \frac{t}{2\mu} = f; \quad D - t + \frac{t}{4} = 2f$$

$$D = 2f + t \left(\frac{1}{\mu} - 1 \right)$$



7.(A) (A) is correct field from right and left are cancelling out



(C) $\frac{\sigma}{\epsilon_0} 2d = 3v$

(D) $\frac{\sigma}{2\epsilon_0} \times d$ (Non zero)

8.(ABC) $\eta_{\text{carnot engine}} = 0.4 = 1 - \frac{T_L(\sin K)}{T_H(\text{source})} \Rightarrow T_{L(\sin k)} = 600K$

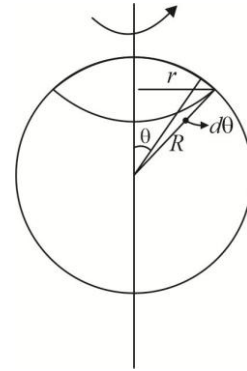
Also $\eta = \frac{\text{Workdone}}{\text{Heat supplied}} = 0.4 = \frac{W}{150} \Rightarrow W = 60J$

Heat pump works on reverse Carnot by the ℓ coefficient of performance $= \frac{T_H}{T_H - T_C} = 10 = \frac{300}{300 - T_C}$

$\Rightarrow T_C = 270K; \text{Cop} = \frac{Q_{out}}{W_{\text{Done on system}}} = 10 = \frac{Q_{out}}{60} = 600J$

9.(1.67) If we assume charges to be uniformly distributed on surface of conductor then

$$\begin{aligned}
 M &= \int dq \times \frac{10}{2\pi} \times \pi r^2 \\
 &= \int \frac{\theta}{4\pi R^2} \times 2\pi r R d\theta \times \frac{\omega}{2\pi} \times \pi r^2 = \frac{\theta\omega}{4R} \int r^3 d\theta \\
 &= \frac{\theta\omega}{4R} \int_0^\pi R^3 \sin^3 \theta d\theta = \frac{\theta\omega R^2}{3}
 \end{aligned}$$



$$L = \frac{2}{5} m R^2 \omega; \quad \frac{M}{L} = \frac{5}{6} \frac{\theta}{m}; \quad \alpha = \frac{5}{3} = 1.67$$

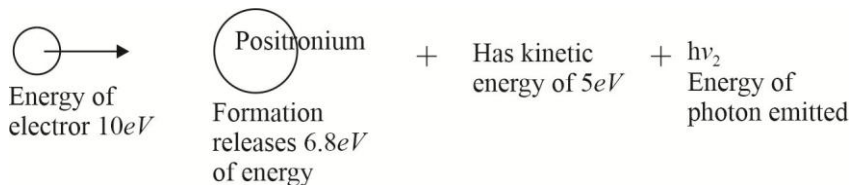
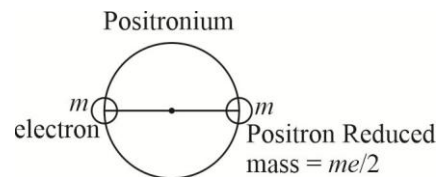
However for a rotating sphere, in addition to electrostatic force of repulsion there will be force due to rotation of sphere and then charge configuration can't be determined.

10.(11.8) By conservation energy.

$$h\nu_1 = 13.6 + 10$$

$$\Rightarrow h\nu_1 = 23.6 \text{ eV}$$

Therefore when compared to energy of electron in hydrogen atom, ground state energy of positronium should be $-\frac{13.6 \text{ eV}}{2} = -6.8 \text{ eV}$



By conservation of energy.

$$10 \text{ eV} = -6.8 \text{ eV} + 5 \text{ eV} + h\nu_2 \Rightarrow h\nu_2 = 11.8 \text{ eV}$$

$$h\nu_1 - h\nu_2 = 23.6 - 11.8 = 11.8 \text{ eV}$$

11.(1.6) xy is an isobaric process

$$Q_{xy} = nC_p R \Delta T = n \times \frac{5}{2} \times R(T_y - T_x)$$

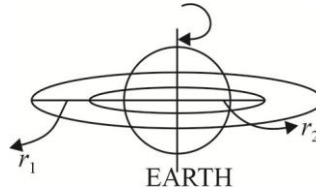
$$\boxed{\frac{T_y}{250} = \frac{T_x}{125} \Rightarrow T_y = 2T_x} = \frac{5}{2} \times nRT_x$$

$$T_w v_w^{\gamma-1} = T_x v_x^{\gamma-1}$$

$$\Rightarrow T_w \times (64)^{2/3} = T_x (125)^{2/3} \Rightarrow \boxed{T_x = T_w \times \frac{16}{25}}$$

$$= \frac{5}{2} \times n \times RT_w \times \frac{16}{25} = \frac{5}{2} \times \frac{16}{25} = 1.6$$

12.(2.33) Given $r_1 = 1.21r_2$



Geostationary Satellite rotates in same sense as Earth

$\Rightarrow$ Geostationary satellite is in radius r_1

While the 2nd satellite is rotating in opposite sense in radius r_2

From keplers law $\omega^2 \propto \frac{1}{r^3}$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow \frac{\omega_2}{\omega_1} = (1.21)^{3/2}; \quad \frac{2\pi}{\omega_1} = 24$$

$$\Rightarrow \omega_2 = (1.1)^3 \times \omega_1$$

$$\Rightarrow \omega_2 = 1.331 \times \omega_1 = 1.331 \times \frac{2\pi}{24} = 1.331 \times \frac{2\pi}{24}$$

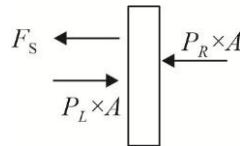
$$T = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{1.331 \times \frac{2\pi}{24} + \frac{2\pi}{24}} = \frac{2\pi}{\frac{2\pi}{24}(2.331)} = \frac{24}{2.331}$$

13.(0.2) Natural length of spring = $0.4L$

Present length of spring = $0.5L$

Elongation in spring = $0.1L$

F.B.D. of piston at equilibrium



$$\Rightarrow F_s + P_R \times A = P_L \times A \Rightarrow F_s + P_R \times A = \frac{3}{2} P_R \times A$$

$$\Rightarrow F_s = \frac{P_R \times A}{2} \Rightarrow P_R = \frac{2}{A} \times K \times 0.1 \times L$$

Since piston is conducting, temperature in both compartment will be same

$$P_R \times v = 1 \times R \times T; \quad P_L \times v = \frac{3}{2} \times R \times T \Rightarrow \frac{P_L}{P_R} = \frac{3}{2} \Rightarrow \boxed{\alpha = 0.2}$$

$$14.(1.2) \tan \theta = \frac{t}{2l + d} = \frac{12 \times 10^{-6}}{2 \times 10^{-3} + 2 \times 10^{-3}} = 3 \times 10^{-3}$$

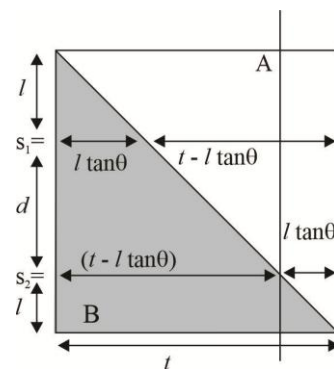
$$\frac{n_A l \tan \theta}{C} + \frac{n_B (t - l \tan \theta)}{C} = t_1$$

$$\frac{n_B l \tan \theta}{C} + \frac{n_A (t - l \tan \theta)}{C} = t_2$$

$$\Delta t = \frac{(n_A - n_B) l \tan \theta + (n_B - n_A) (t - l \tan \theta)}{C};$$

$$d \sin \theta = \frac{dY}{D}$$

$$\Rightarrow \Delta x = \frac{(n_A - n_B)(2l \tan \theta - t)}{C} = \frac{dY}{D} \Rightarrow \Delta x = \frac{(n_A - n_B)(2l \tan \theta - t) \times D}{Cd} = Y$$



15.(170) $v_0 = 270$

$$v_{0x} = 270 \cos 60 = 270 \times \frac{1}{2} = 135$$

At any intermediate time

$$-m\vec{g} - C\vec{v}_y = \frac{m d\vec{v}_y}{dt} \quad \dots (1)$$

$$-C\vec{v}_x = \frac{m d\vec{v}_x}{dt} \quad \dots (2)$$

$$\Rightarrow \int \frac{dv_x}{v_x} = \frac{-C}{m} \int dt \quad = \ln v_x \Big|_{135}^{v_x} = -\frac{0.1}{0.2} \times 2 = \ln \frac{135}{v_x} = 1$$

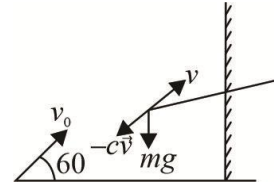
$$\Rightarrow \left(\frac{135}{v_x} \right)^1 = e = 2.7 \quad \Rightarrow \quad v_x = \frac{135}{2.7}$$

Rewriting equation (2) $-C\vec{v}_x = m \frac{v_x dv_x}{dx}$

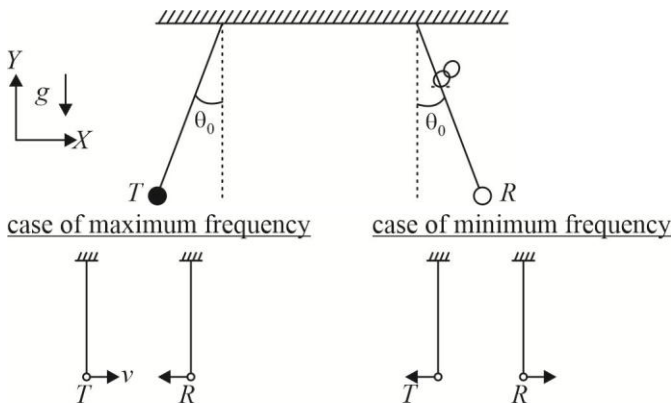
$$\Rightarrow \int dx = -\frac{m}{c} \int \frac{v_x dv_x}{v_x} \quad \Rightarrow \quad x = \frac{m}{c} \left[135 - \frac{135}{2.7} \right]$$

$$\Rightarrow x = \frac{0.2}{0.1} \frac{(135)(2.7-1)}{2.7}$$

$$\Rightarrow x = 2 \times 135 \times \frac{1.7}{2.7} = 270 \times \frac{17}{27} = 170$$



16.(32)



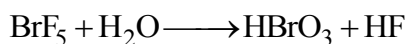
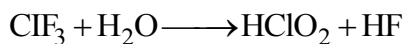
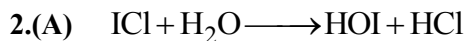
$$f_{\max} = f_0 \left(\frac{330+v}{330-v} \right) = f_0 \quad f_{\min} = f_0 \left(\frac{330-v}{330+v} \right) = f_0 x$$

$$v = \sqrt{2gh} = \sqrt{2g\ell(1-\cos\theta)} = \sqrt{2 \times 10 \times 8 \times (1-0.9)} = 4$$

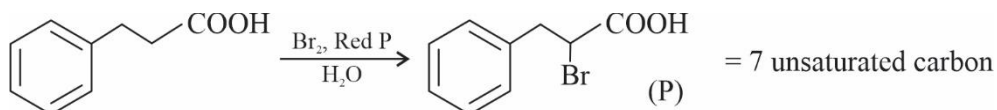
$$\Delta f_{\max} = f_0 \left\{ \frac{334}{326} - \frac{326}{334} \right\} = 660[1.0245 - 0.9760]$$

$$= 660 \times 0.0485 = 32.01 \simeq 32$$

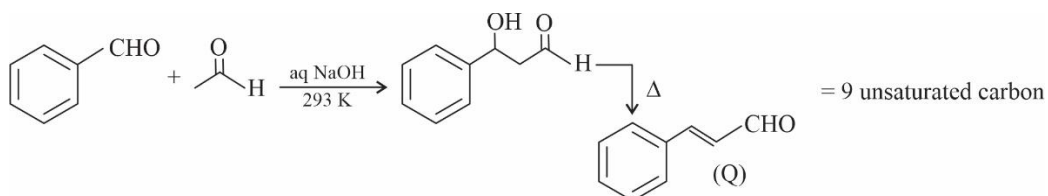
CHEMISTRY



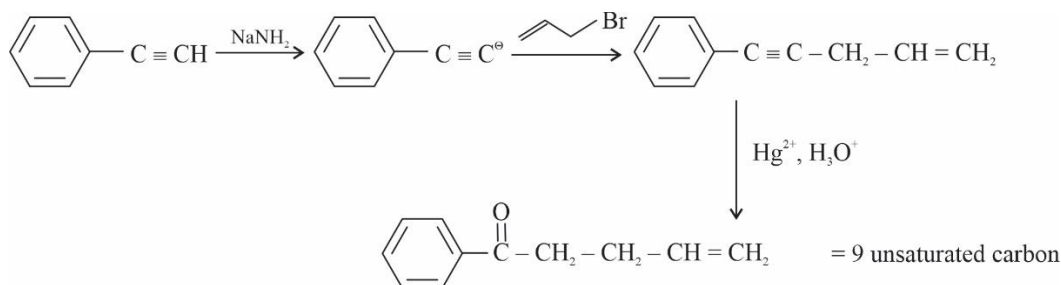
3.(D) (i)



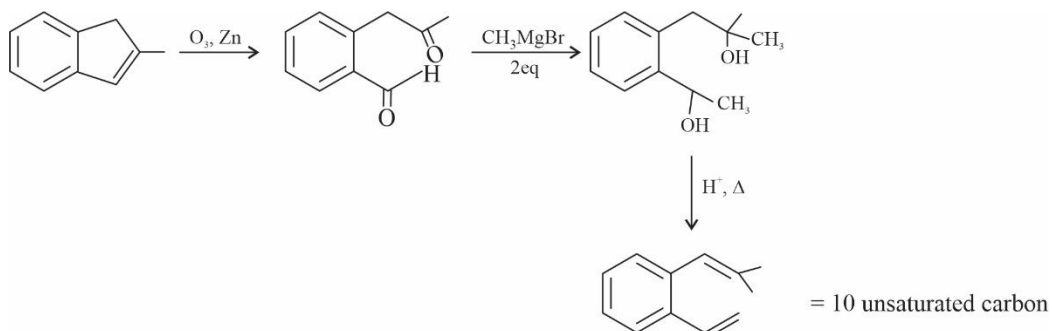
(ii)



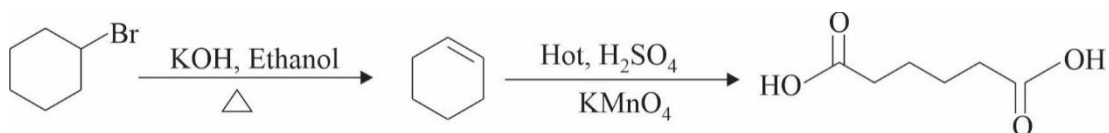
(iii)



(iv)



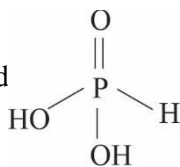
4.(C)



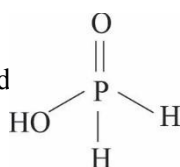
5.(CD) Statement A and B are incorrect.

- The potential energy between a point dipole and a point charge approaches zero more rapidly as $\left(PE \propto \frac{1}{r^2}\right)$ compared to that between two point charges where $\left(PE \propto \frac{1}{r}\right)$
- The average potential energy of two rotating polar molecules that are separated by a distance r has $\frac{1}{r^6}$ dependence.
- The dipole induced dipole interaction energy is independent of temperature because thermal motion has no effect on averaging process.

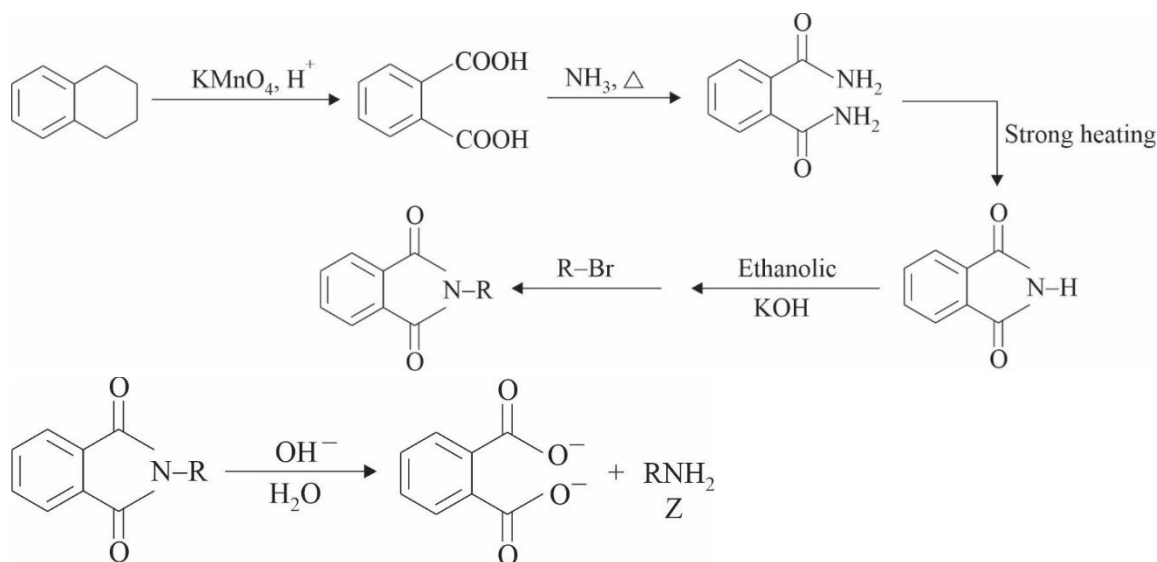
6.(BD) H_3PO_3 : has one P – H bond



H_3PO_2 : has two P – H bond



7.(AC)



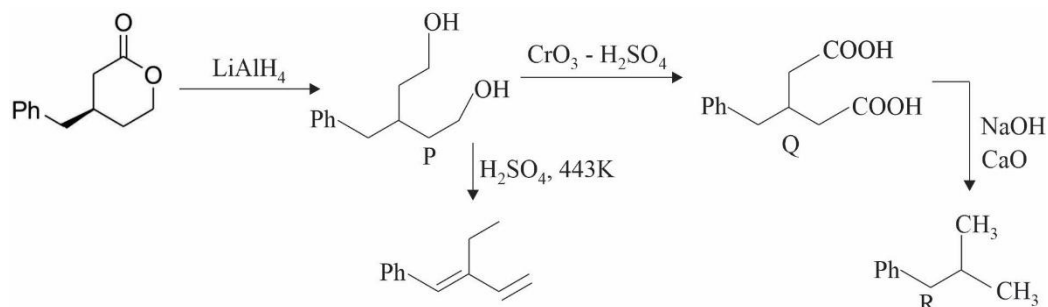
(A) is correct

(B) Phthalimide does not form isocyanide with $CHCl_3 / KOH$

(C) 1° amine reacts with Hinsberg reagent

(D) Incorrect

8.(BC)

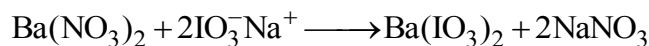


BC are correct.

$$9.(11) \quad d = \frac{Z \times M}{N_A \times a^3}$$

$$d = \frac{4 \times 105.6}{6 \times 10^{23} \times (400 \times 10^{-12})^3} = 1.1 \times 10 = 11$$

10.(3.95)



$$200 \text{ ml} \quad 100 \text{ ml}$$

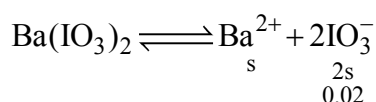
$$0.01 \text{ M} \quad 0.1 \text{ M}$$

$$2 \text{ m mol} \quad 10 \text{ m mol}$$

$$\quad \quad \quad 6 \text{ m mol} \quad 2 \text{ m mol} \quad 4 \text{ m mol}$$

$$\text{IO}_3^- \text{ left} = 6 \text{ m mol}$$

$$[\text{IO}_3^-] = \frac{6}{300} = 0.02 \text{ M}$$



$$K_{sp} = s(2s + 0.02)^2$$

$$1.58 \times 10^{-9} = s(0.02)^2$$

$$s = 3950 \times 10^{-9} = 3.950 \times 10^{-6}$$

$$11.(16) \quad \frac{x}{m} = KC^{\frac{1}{n}}$$

$$\log \frac{x}{m} = \log K + \frac{1}{n} \log C$$

$$\log 4 = \log K + \frac{1}{n} \log 10 \quad \dots(i)$$

$$\log 10 = \log K + \frac{1}{n} \log 16 \quad \dots(ii)$$

$$(ii - i)$$

$$\log 10 - \log 4 = \frac{1}{n} (2 \log 4 - \log 10)$$

$$\frac{1}{n} = \frac{1 - \log 4}{2 \log 4 - 1}$$

$$\frac{1}{n} = \frac{1 - 0.6}{1.2 - 1} = 2$$

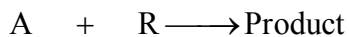
$$\log \frac{x}{m} = \log K + \frac{1}{n} \log 20 = \log K + \frac{1}{n} (\log 2 + \log 10)$$

$$= \log K + \frac{1}{n} + \frac{1}{n} \log 2 \quad \left(\frac{1}{n} = 2 \right)$$

$$= \log 4 + 2 \log 2$$

$$\log \frac{x}{m} = \log 16; \quad \frac{x}{m} = 16$$

12.(4.17)



$$A_o \quad 10A_o$$

$$A_o - x \quad 10A_o - x \quad x$$

$$\text{Given } x = 0.4$$

$$A_t = 0.6 A_o \quad R_t = 9.6 A_o$$

$$\text{Actual rate} = k(A_t)(R_t)$$

$$\text{Actual rate} = k(0.6 A_o)(9.6 A_o)$$

$$\text{Rate} = 5.76 k A_o^2$$

$$\text{As per assumption ; } \text{rate} = k^1[A]$$

$$\text{rate} = k[R_o][A] = k[10 A_o][0.6 A_o]$$

$$\text{Pseudo rate} = 6k A_o^2$$

$$\text{Relative error} = \frac{6 - 5.76}{5.76} \times 100 = 4.166 = 4.17$$

13.(2.49) $\pi = h\rho g = MRT$

$$\pi = \frac{2}{100} \times (1 \times 1000) \times 10 \quad \left(h = 0.02 \text{ m ; } d = 1000 \text{ kg m}^{-3} \right)$$

$$= 200 \text{ Pa}$$

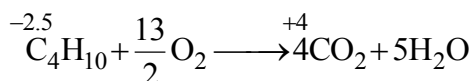
$$200 = MRT$$

$$M = \frac{200}{RT} = \frac{200}{8.3 \times 300} = 0.0803 \text{ mol m}^{-3} = 0.0803 \times 10^{-3} \text{ mol L}^{-1}$$

$$\text{Given: Strength} = 2 \text{ g L}^{-1} = \text{Molarity} \times \text{Molar Mass}$$

$$\text{Molar mass} = \frac{2}{0.0803 \times 10^{-3}} = 24906.6002 = 2.49 \times 10^4$$

14.(105.5)



$$n_f = 6.5 \times 4 = 26$$

$$\Delta G_f^\circ = 4\Delta G_f^\circ CO_2 + 5\Delta G_f^\circ H_2O - \Delta G_f^\circ C_4H_{10}$$

$$= 4(-394) + 5(-237) - (-18) = -2743 \text{ kJ mol}^{-1} = -2743 \times 10^3 \text{ J mol}^{-1}$$

$$\Delta G^\circ = -nFE^\circ_{\text{cell}}$$

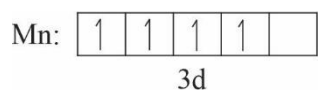
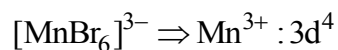
$$E^\circ_{\text{cell}} = -\frac{\Delta G^\circ}{nF} \quad (n = 26)$$

$$= -\frac{2743 \times 1000}{26 F}$$

$$\frac{x}{F} \times 10^3 = \frac{105.5 \times 10^3}{F}$$

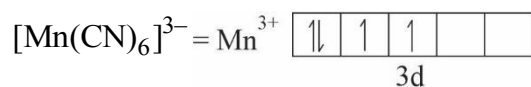
$$x = 105.5$$

15.(7.73)



: 4 unpaired electron

$$\mu = \sqrt{4(4+2)} = \sqrt{24}$$



$$\mu = \sqrt{2(2+2)} = \sqrt{8}$$

$$\mu_T = \sqrt{24} + \sqrt{8} = 7.727 = 7.73$$

16.(2) Octasaccharide + $7\text{H}_2\text{O} \longrightarrow \text{Ribose} + \text{de-oxy Ribose} + \text{Glucose}$

Let number of ribose be x, 2-deoxyribose be y, glucose = z units

$$x + y + z = 8 \quad \dots(i)$$

$$150x + 134y + 180z = 1024 + 18(7) \quad \dots(ii)$$

$$150x + 134y + 180z = 1150$$

$$\frac{134y}{1150} = 0.5826$$

$$y \approx 5$$

Putting y in (ii)

$$150x + 180z = 480$$

$$5x + 6z = 16$$

Putting y in (i) $x + z = 3$

Solving it ; $z = 1$

$$x = 2$$

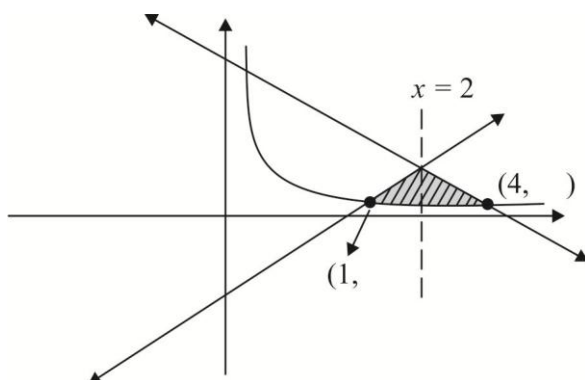
1.(C) Using LH and then replacing $e^{x_0} = -x_0$

And $\alpha = 3$

$\ell = 0$

2.(B) Req. Area $\int_1^2 \frac{5x-1}{4} - \frac{1}{x} dx + \int_2^4 \frac{17-4x}{4} - \frac{1}{x} dx$

$$= \frac{33}{8} - \ln 4$$



3.(C) $\theta = \tan^{-1}(2 \tan \theta) - \frac{1}{2} \sin^{-1} \left(\frac{6 \tan \theta}{9 + \tan^2 \theta} \right)$

Let $\frac{1}{3} \tan \theta = x$

$$\tan^{-1}(3x) = \tan^{-1}(6x) - \frac{1}{2} \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$\tan^{-1}(3x) = \tan^{-1}(6x) - \tan^{-1} x$$

$$3x = \left[\frac{6x-x}{1+6x^2} \right]$$

$$3x(1+6x^2) - 5x = 0$$

$$x[3+18x^2-5] = 0$$

$$\therefore x = 0, \pm \frac{1}{3}$$

$$\therefore \theta = 0, \pm \frac{\pi}{4}$$

4.(A) Eliminating ' α '

Locus of P.O.I is

$$S: \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\text{Slope of tangent} \Rightarrow m = \frac{4\sqrt{2}}{3}$$

Equation of tangent :

$$\Rightarrow y = \frac{4\sqrt{2}}{3}x + 4 \text{ passing through } \left(-\frac{3}{\sqrt{2}}, 0\right) \text{ and } (0, 4) \therefore pq = -6\sqrt{2}$$

5.(AB) $QR = RP$

$$\begin{bmatrix} x & y \\ z & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} ax + cy & bx + dy \\ az + 4c & bz + 4d \end{bmatrix} = \begin{bmatrix} 2a & 3b \\ 2c & 3d \end{bmatrix}$$

$$\text{Now } ax + cy = 2a \quad \dots(i)$$

$$bx + dy = 3b \quad \dots(ii)$$

$$az + 4c = 2c \Rightarrow -az = 2c$$

$$bz + 4d = 3d \Rightarrow -bz = d$$

From (i)

$$ax - \frac{az}{2}y = 2a \Rightarrow 2x - yz = 4 \quad \dots(iii)$$

From (ii)

$$bx - bzy = 3b$$

$$x - yz = 3 \quad \dots(iv)$$

Solving (iii) and (iv)

$$x = 1 \text{ and } yz = -2$$

$$(A) \quad |Q - 2I| = \begin{vmatrix} x-2 & y \\ z & 2 \end{vmatrix} = 2x - 4 - yz = 0 \quad (B) \quad |Q - 6I| = \begin{vmatrix} x-6 & y \\ z & -2 \end{vmatrix} = -2x + 12 - yz = 12$$

$$(C) \quad |Q - 3I| = \begin{vmatrix} x-3 & y \\ z & 1 \end{vmatrix} = x - 3 - yz = 0 \quad (D) \quad yz = -2$$

6.(AC) $y^2 = x$

Equation of chord with a given mid point

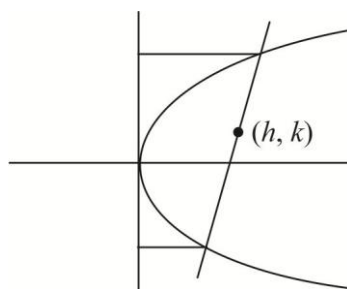
$$T = S_1$$

$$y \cdot k - \frac{(x+h)}{2} = k^2 - h$$

$$2ky - x - h = 2k^2 - 2h$$

$$x = 2ky - 2k^2 + h$$

$$\int_{y_1}^{y_2} (2ky - 2k^2 + h) - y^2 dy = \frac{4}{3}$$



$$\Rightarrow ky^2 + (h - 2k^2)y - \frac{y^3}{3} \Big|_{y_1}^{y_2} = \frac{4}{3} \Rightarrow ky^2 + (h - 2k^2)y - \frac{y^3}{3} \Big|_{y_1}^{y_2} = \frac{4}{3}$$

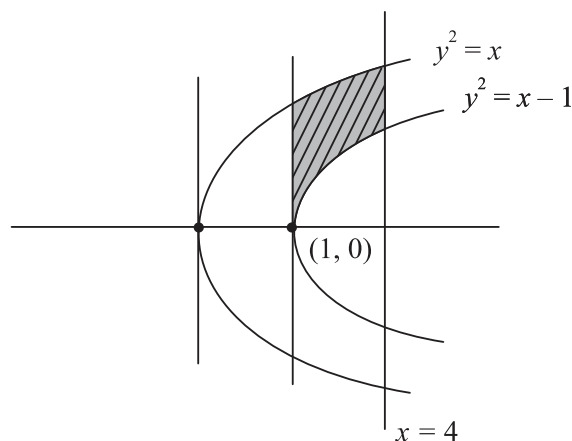
Solving chord and parabola

$$y^2 - 2ky + 2k^2 - h = 0$$

$$y_1 + y_2 = 2k$$

$$y_1 y_2 = 2k^2 - h$$

Locus of mid-point is $\Rightarrow x - y^2 = 1$



$$\text{Req. Area} = \int_1^4 \sqrt{x} - \sqrt{x-1} dx \Rightarrow \frac{2}{3} x^{3/2} - \frac{2}{3} (x-1)^{3/2} \Big|_1^4 \Rightarrow \frac{14}{3} - 2\sqrt{3} \quad (\text{C})$$

7.(AC) $\because$ R lies on circle

$$x_1^2 + \frac{x_1^2}{3} = 9$$

$$\Rightarrow x_1 = \frac{3\sqrt{3}}{2}$$

$$\therefore R\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right) \text{ and } P\left(\frac{3\sqrt{3}}{2}, 1\right)$$

$\because$ S lies on circle

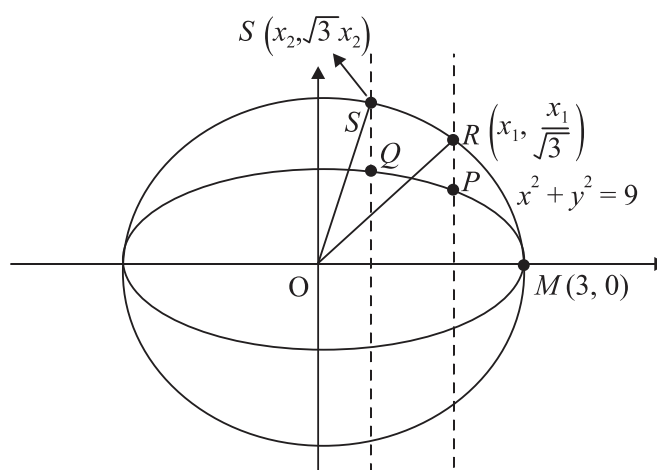
$$x_2^2 + 3x_2^2 = 9$$

$$\Rightarrow x_2 = \frac{3}{2}$$

$$\therefore S\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right) \text{ and } Q\left(\frac{3}{2}, \sqrt{3}\right)$$

Equation of line joining P and Q is

$$y - 1 = \frac{\sqrt{3} - 1}{\frac{3}{2} - \frac{3\sqrt{3}}{2}} \left(x - \frac{3\sqrt{3}}{2} \right) \Rightarrow 2x + 3y = 3(1 + \sqrt{3}) \quad (\text{A})$$



$$N_2 = (x_2, 0)$$

$$N_2 = \left(\frac{3}{2}, 0 \right)$$

$$Q = \left(\frac{3}{2}, \sqrt{3} \right)$$

$$S = \left(\frac{3}{2}, \frac{3\sqrt{3}}{2} \right)$$

$$|N_2 S| = \frac{3\sqrt{3}}{2} \text{ and } |N_2 Q| = \sqrt{3}$$

$$\therefore |N_2 S| = \frac{3}{2} |N_2 Q| \quad \Rightarrow \quad 2|N_2 S| = 3|N_2 Q| \quad \text{and} \quad N_1 = (x_1, 0)$$

$$N_1 = \left(\frac{3\sqrt{3}}{2}, 0 \right)$$

$$P = \left(\frac{3\sqrt{3}}{2}, 1 \right)$$

$$R = \left(\frac{3\sqrt{3}}{2}, \frac{3}{2} \right)$$

$$|N_1 P| = 1$$

$$|N_1 R| = \frac{3}{2} \quad \Rightarrow \quad 2|N_1 R| = 3|N_1 P|$$

8.(BCD) $f(x) = \frac{6x + \sin x}{2x + \sin x}; x \neq 0$

$f(x)$ is continuous

$$f'(x) = \frac{(2x + \sin x)(6 + \cos x) - (6x + \sin x)(2 + \cos x)}{(2x + \sin x)^2}$$

$$f'(x) = \frac{4(\sin x - x \cos x)}{(2x + \sin x)^2} = 0$$

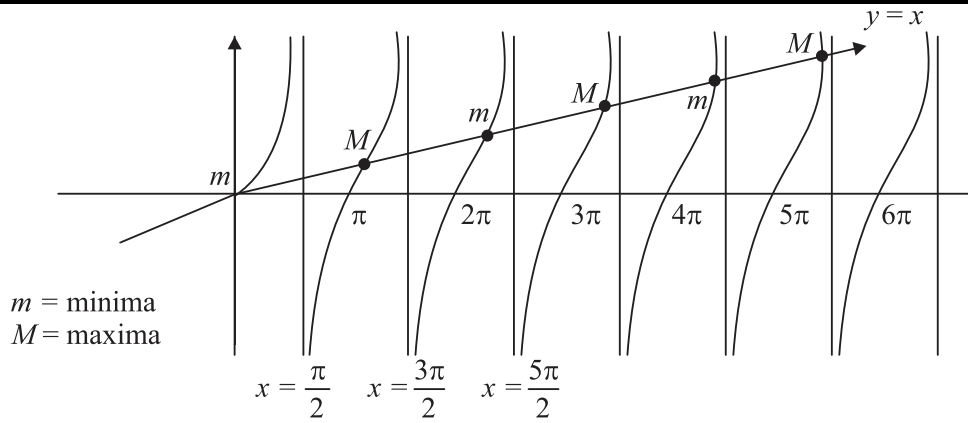
$$f'(x) = 4 \cos x (\tan x - x) = 0$$

$$\tan x = x \Rightarrow x = 0$$

$$f'(0^+) = +ve$$

$$f'(0^-) = -ve$$

$\therefore$ Local minima at $x = 0$



$$9.(0.75) \quad x^2 \frac{dy}{dx} + xy = x^2 + y^2 \Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2 - xy}{x^2}$$

$$\text{Let } y = vx \Rightarrow v + x \frac{dv}{dx} = 1 + v^2 - v \Rightarrow x \frac{dv}{dx} = 1 + v^2 - 2v$$

$$\Rightarrow \int \frac{dv}{(v-1)^2} = \int \frac{dx}{x} \Rightarrow \frac{-1}{v-1} = \ln|x| + C$$

$$\Rightarrow \frac{x}{x-y} = \ln|x| + C \Rightarrow \frac{x}{x-y} = \ln x + C \quad (\because x > 1/e)$$

$$\text{Given: } x=1 \Rightarrow y=0 \Rightarrow c=1$$

$$\Rightarrow \frac{x}{x-y} = \ln x + 1 \Rightarrow y(e) = \frac{e}{2} \text{ and } y(e^2) = \frac{2e^2}{3}$$

$$\Rightarrow \frac{2(y(e))^2}{y(e^2)} = \frac{2 \cdot e^2}{4 \cdot 2e^2} \cdot 3 = \frac{3}{4} = 0.75$$

$$10.(6) \quad \frac{T_{r+1}}{T_r} = \frac{{}^{23}C_r \left(\frac{2}{5}\right)^r}{{}^{23}C_{r-1} \left(\frac{2}{5}\right)^{r-1}}$$

$$\frac{T_{r+1}}{T_r} = \frac{48-2r}{5r}$$

$$T_7 > T_6 \text{ and } T_8 < T_7$$

$$\therefore T_7 \text{ will have largest coefficient } \therefore a_6 \text{ will be largest } \therefore r = 6$$

11.(0.30)

$A \rightarrow$ bulb is defective

$M_1 \rightarrow$ bulb is produced by M_1

$M_2 \rightarrow$ bulb is produced by M_2

$M_3 \rightarrow$ bulb is produced by M_3

$$P(M_1) = \frac{2}{5}, P(M_2) = \frac{2}{5}, P(M_3) = \frac{1}{5}$$

$$P\left(\frac{A}{M_1}\right) = \frac{3}{20}$$

$$P(A) = \frac{2}{5} \cdot \frac{3}{20} + P\left(\frac{A}{M_2}\right) \cdot \frac{2}{5} + P\left(\frac{A}{M_3}\right) \cdot \frac{1}{5}$$

$$\Rightarrow 2P\left(\frac{A}{M_2}\right) + P\left(\frac{A}{M_3}\right) = \frac{7}{10} \quad \dots(i)$$

$$\text{Now } P\left(\frac{M_2}{A}\right) = \frac{\frac{2}{5}P\left(\frac{A}{M_2}\right)}{\frac{1}{5}} = \frac{2}{5} \Rightarrow P\left(\frac{A}{M_2}\right) = \frac{1}{5}$$

From (i)

$$\Rightarrow P\left(\frac{A}{M_3}\right) = \frac{3}{10}$$

$$12.(-2) \quad \vec{x} = (\alpha + 2\beta - 3)\hat{i} + (3\alpha + 3\beta - 1)\hat{j} + (3\alpha + \beta - 2)\hat{k}$$

$$\vec{y} = (2\alpha + 3\beta - 1)\hat{i} + (3\alpha + \beta - 2)\hat{j} + (\alpha + 2\beta - 3)\hat{k}$$

$$\vec{z} = (3\alpha + \beta - 2)\hat{i} + (\alpha + 2\beta - 3)\hat{j} + (2\alpha + 3\beta - 1)\hat{k}$$

$$\therefore \quad \vec{x}, \vec{y} \text{ and } \vec{z} \text{ are coplanar} \quad \therefore \quad \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = 0$$

$$(x + y + z) \begin{vmatrix} 1 & 1 & 1 \\ y & z & x \\ z & x & y \end{vmatrix} = 0$$

$$(\alpha + 2\beta - 3 + 2\alpha + 3\beta - 1 + 3\alpha + \beta - 2) = 0$$

$$6\alpha + 6\beta - 6 = 0$$

$$\alpha + \beta = 1$$

$$\alpha + \beta - 3 = -2$$

$$13.(-2) \quad \sum_{n=1}^{2025} (-\omega)^n \Rightarrow \frac{(-\omega)[1 - (-\omega)^{2025}]}{1 - (-\omega)}$$

$$\Rightarrow \frac{(-\omega)[1 + \omega^{2025}]}{1 + (\omega)} = \frac{-2\omega}{1 + \omega} = \frac{-2\omega}{-\omega^2} \Rightarrow \frac{2}{\omega} \Rightarrow 2\omega^2 \Rightarrow \arg(2\omega^2) \Rightarrow -\frac{2\pi}{3} = \alpha$$

$$\text{Now } \frac{3\alpha}{\pi} = -2$$

$$14.(0.25) \quad g(x) = \frac{4}{1+e^{-2x}} = y$$

$$\Rightarrow -\frac{1}{2} \ln \left(\frac{4}{x} - 1 \right) = g^{-1}(x) = h(x)$$

$$\text{To find } D(f(h(x))) \Rightarrow f'(h(x)) \cdot h'(x)$$

$$\Rightarrow f'(h(2)) \times -\frac{1}{2} \cdot \frac{1}{\frac{4}{x}-1} \cdot -\frac{4}{x^2} \Rightarrow f'(0) \times \frac{2}{\left(\frac{4}{2}-1\right) \times 4}$$

$$\Rightarrow \frac{1}{2} f'(0) \Rightarrow \frac{1}{2} \cdot \frac{1}{x^2+2x+4} \times 2x+2 \text{ at } x=0 \Rightarrow 0.25$$

$$15.(3) \quad T_1 = \frac{1}{\sin 1^\circ} \cdot \frac{\sin(61^\circ - 60^\circ)}{\sin 60^\circ \sin 61^\circ}; \quad T_1 = \frac{1}{\sin 1^\circ} [\cot 60^\circ - \cot 61^\circ]$$

$$T_2 = \frac{1}{\sin 1^\circ} [\cot 64^\circ - \cot 65^\circ]; \quad T_{n-1} = \frac{1}{\sin 1^\circ} [\cot 116^\circ - \cot 117^\circ]$$

$$T_n = \frac{1}{\sin 1^\circ} [\cot 118^\circ - \cot 119^\circ] \therefore \alpha = \frac{1}{\sin 1^\circ} [\cot 60^\circ + \cot 90^\circ] \therefore \alpha = \frac{1}{\sqrt{3}} \operatorname{cosec} 1^\circ$$

$$\text{Now } \left(\frac{\operatorname{cosec} 1^\circ}{\alpha} \right)^2 \Rightarrow \left(\frac{\operatorname{cosec} 1^\circ}{\operatorname{cosec} 1^\circ} \cdot \sqrt{3} \right)^2 = 3$$

$$16.(21) \quad \int_{1/2}^2 \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx = I$$

$$\text{Let } x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$$

$$\Rightarrow I = \int_2^{1/2} \frac{\tan^{-1} \left(\frac{1}{t} \right) \cdot -\frac{1}{t^2} dt}{\frac{2t^2 - 3t + 2}{t^2}} \Rightarrow I = \int_{1/2}^2 \frac{\tan^{-1} \left(\frac{1}{x} \right) dx}{2x^2 - 3x + 2}$$

$$\Rightarrow 2I = \frac{\pi}{2} \int_{1/2}^2 \frac{dx}{2x^2 - 3x + 2} \Rightarrow I = \frac{\pi}{8} \int_{1/2}^2 \frac{dx}{x^2 - \frac{3}{2}x + 1} = \frac{\pi}{8} \cdot \frac{4}{\sqrt{7}} \tan^{-1} \frac{(4x-3)}{\sqrt{7}}$$

$$\Rightarrow \frac{\pi}{2\sqrt{7}} \left[\tan^{-1} \frac{5}{\sqrt{7}} + \tan^{-1} \frac{1}{\sqrt{7}} \right] \Rightarrow \frac{\pi}{2\sqrt{7}} \tan^{-1} (3\sqrt{7}) = \alpha$$

$$\text{Now } \sqrt{7} \tan \left(\frac{2\alpha\sqrt{7}}{\pi} \right) = \sqrt{7} \tan \left(\frac{2\sqrt{7}}{\pi} \cdot \frac{\pi}{2\sqrt{7}} \tan^{-1} (3\sqrt{7}) \right) = 21$$