SOLUTIONS OF JEE ADVANCED 2025 | PAPER - 1

PHYSICS

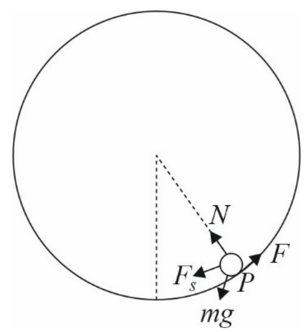
1.(A) Torque equation about point of contact $\tau_P = I_P \alpha$

$$F_s r + mg \sin\theta r = \left(\frac{3}{2}mr^2\right)\frac{a}{r}$$

$$kxr + mg\frac{x}{R-r}r = \frac{3}{2}mra$$

$$a = \frac{2}{3}\left(\frac{k}{m} + \frac{g}{R-r}\right)x$$

$$\therefore \quad \omega^2 = \frac{2}{3}\left(\frac{k}{m} + \frac{g}{R-r}\right)$$



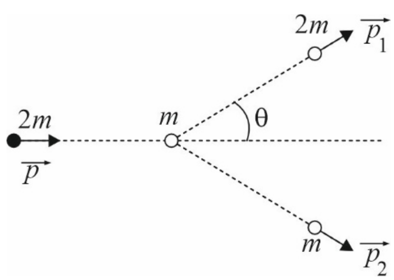
2.(D) Momentum conservation

$$\vec{p}_1 + \vec{p}_2 = \vec{p} \quad \Rightarrow \quad \vec{p}_2 = \vec{p} - \vec{p}_1 \quad \therefore \quad p_2 = \sqrt{p^2 + p_1^2 - 2pp_1\cos\theta}$$

Energy conservation

$$\frac{p_1^2}{4m} + \frac{p_2^2}{2m} = \frac{p^2}{4m}$$

$$p_1^2 + 2\left(p^2 + p_1^2 - 2pp_1\cos\theta\right) = p^2$$



$$3p_1^2 - (4p\cos\theta)p_1 + p^2 = 0$$

For real root of $p_1, D \geq 0$

$$\Rightarrow \quad 16p^2\cos^2\theta - 12p^2 \geq 0 \quad \Rightarrow \quad \cos^2\theta \geq \frac{3}{4}$$

$$\Rightarrow \quad \cos\theta \geq \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta \leq \frac{\pi}{6}$$

3.(A) After time t, $\vec{A} = a^2\left(-\cos\omega t\,\hat{j} + \sin\omega t\,\hat{k}\right)$

$$\therefore \quad \Phi = \vec{B}\cdot\vec{A} = B_0 a^2 \sin\omega t\cos\omega t = \frac{B_0 a^2}{2}\sin 2\omega t$$

$$V = -\frac{d\phi}{dt} = \frac{-B_0 a^2}{2} 2\omega\cos 2\omega t = -B_0 a^2 \omega\cos 2\omega t \qquad \text{For } 0 < t < \frac{\pi}{\omega}$$

$$V = 0 \quad \text{For } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$$

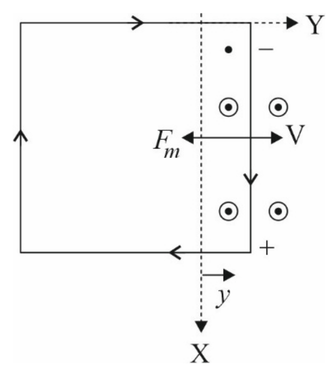
4.(C) $1\,MSD = 0.1\,cm$ $\qquad V = 0$

$10\,VSD = 7\,MSD \rightarrow 1USD = 0.7\,cm$

$LC = 1MSD - 1VSD = 0.03\,cm$

$D = 0.1\,cm + 0.03 = 0.13\,cm$

5.(BD) Let velocity of loop be v after it moves a distance y.



$$\varepsilon_{induced} = B_0 L v$$

$$I = \frac{B_0 L v}{R}$$

$$F_m = B_0 I L = \frac{B_0^2 L^2}{R} v$$

$$a = \frac{-B_0^2 L^2 V}{MR} = \frac{dv}{dt} \Rightarrow dv = -Kvdt$$

$$\int_{v_0}^{v} \frac{dv}{v} = -K\int_0^t dt \Rightarrow v = v_0 e^{-Kt}$$

$$\frac{dy}{dt} = v_0 e^{-Kt} \Rightarrow \int_0^y dy = v_0 \int_0^t e^{-Kt} dt \quad \therefore \quad y = \frac{v_0}{K}\left(1 - e^{-Kt}\right)$$

(A) If $v_0 = 1.5KL$

$y_{\max} = \frac{v_0}{K} = 1.5L > L \qquad \therefore \qquad$ Loop will enter field completely

(B) When complete loop is inside, $I = 0 \quad \Rightarrow F = 0$

(C) If $v_0 = \frac{KL}{10}$

$$y_{\max} = \frac{v_0}{K} = \frac{L}{10}$$

So, loop stops after travelling $\frac{L}{10}$ but it will take infinite time.

(D) If $v_0 = 3KL$

$$y_{max} = \frac{v_0}{K} = 3L$$

For $y = L$,

$$L = 3L\,(1 - e^{-Kt}) \Rightarrow e^{-Kt} = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow -Kt = ln\frac{2}{3} \Rightarrow t = \frac{1}{K}ln\frac{3}{2}$$

6.(D) $l = 10.5\ cm$ (3 significant figures)

$b = 0.05\ mm = 0.05 \times 10^{-1}\ cm$ (1 significant figure)

$t = 6.0 \mu m = 6.0 \times 10^{-4} cm$ (2 significant figure)

$= 3 \times 10^{-5} cm^3$

Rounding off to one significant figure

7.(AD) $v_1 = \sqrt{\frac{T}{\mu}}, v_2 = \sqrt{\frac{T}{4\mu}} = \frac{v_1}{2}, v_3 = \sqrt{\frac{T}{16\mu}} = \frac{v_1}{4}$

$$K_1 = \frac{\omega}{V_1} = K, K_2 = \frac{\omega}{V_2} = 2K, K_3 = \frac{\omega}{V_3} = 4K$$

8.(2) $V = \frac{dy}{dt} = 8\left(\frac{2\pi}{T}\right)\cos\frac{2\pi t}{T}$

$$a = \frac{dV}{dt} = -8\left(\frac{2\pi}{T}\right)^2 \sin\frac{2\pi t}{T}$$

Acceleration amplitude, $a_0 = 8\left(\frac{2\pi}{T}\right)^2 = 8\left(\frac{2\pi}{40\pi}\right)^2 = \frac{1}{50}$

Maximum weight observed $= m(g + a_0)$

Minimum weight observed $= m(g - a_0)$

So, maximum variation $= 2m\,a_0 = 2(50)\left(\frac{1}{50}\right) = 2N$

9.(22.95 – 23.00)

$$\frac{nhv}{1} = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

$$35 \times 10^7 \times 6 \times 10^{-34} \times 10^{15} = \frac{B_0^2}{2 \times 4 \times \frac{22}{7} \times 10^{-7}}$$

$$B_0^2 = 35 \times 6 \times 8 \times \frac{22}{7} \times 10^{-19} = 528 \times 10^{-18}$$

$$B_0 = \sqrt{528} \times 10^{-9} = 22.98 \times 10^{-9} T$$

If $\sqrt{528}$ is takes as $\sqrt{529}$, then α ranges 22.95 to 23.00.

10.(3) For figure 1, $W_0 = \sigma A T_P^4 - \sigma A T_Q^4 = \sigma A\left(T_P^4 - T_Q^4\right)$

For figure 2, Let T_1 and T_2 be the steady state temperature of the two middle plates.

Power transferred between adjacent plates is same for P to 1, 1 to 2 and 2 to Q.

$W_S = \sigma A T_P^4 - \sigma A T_1^4$

$W_S = \sigma A T_1^4 - \sigma A T_2^4$

$W_S = \sigma A T_2^4 - \sigma A T_Q^4$

$\therefore \quad 3W_S = \sigma A(T_P^4 - T_Q^4) = W_0 \quad \therefore \quad \dfrac{W_0}{W_s} = 3$

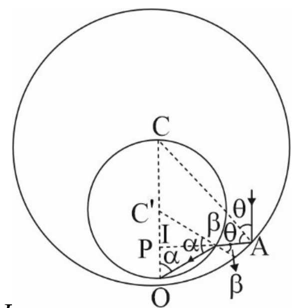
11.(0.75) For polarisation at O,

$\tan\alpha = \sqrt{3} \Rightarrow \alpha = 60°$

Applying shells law

$\sqrt{3}\sin\beta = 1\sin\alpha = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\beta = \dfrac{1}{2} \Rightarrow \beta = 30°$



So, after reflection, path of ray will be shown $A \to B \to O$.

If we extend *AB* to *CO*, it will intersect at P at $90°$ $\quad \therefore \quad C'P = \dfrac{R}{4}$

In ΔCAP, $\sin\theta = \dfrac{CP}{CA} = \dfrac{\frac{R}{2}+\frac{R}{4}}{R} = \dfrac{3}{4} = 0.75$

12.(75.6) $\dfrac{bd}{D} = m\lambda$

$b = \dfrac{m\lambda D}{d} = \dfrac{3\times 600\times 10^{-9}\times 1}{5\times 10^{-3}} = 36\times 10^{-5}$

$\dfrac{\Delta b}{b} = \dfrac{\Delta D}{D} + \dfrac{\Delta d}{d} = \dfrac{10^{-2}}{1} + \dfrac{10^{-3}}{5\times 10^{-3}} = 0.01 + 0.2$

$\Delta b = 0.21\times 36\times 10^{-5} = 75.6\times 10^{-6}$

$\Delta b = 75.6\,\mu m$

13.(72) In $n = 3$,

$L = \dfrac{3h}{2\pi}$ *and* $r = a_0\dfrac{3^2}{Z} \quad \Rightarrow \quad m_e v = \dfrac{L}{r} = \dfrac{3h}{2\pi}\dfrac{z}{9a_0} = \dfrac{hZ}{6\pi a_0} \quad \Rightarrow \quad \dfrac{h}{m_e v} = \dfrac{6\pi a_0}{Z}$

de-Broglie wavelength of electron

$\lambda_n = \dfrac{h}{\sqrt{2m_N K_B T}}$

For $\lambda_n = \lambda_e$

$$\frac{h}{\sqrt{2m_N K_B T}} = \frac{6\pi a_0}{Z} \quad \Rightarrow \quad T = \frac{h^2 Z^2}{72\pi^2 a_0^2 m_N k_B} \quad \therefore \quad \alpha = 72$$

14.(C) **(P)** $\vec{E} = \frac{1}{4\pi \in_0}\frac{p}{r^3}(-\hat{j}) + \frac{1}{4\pi \in_0}\frac{p}{r^3}(-\hat{j}) \Rightarrow P \to 2$

(Q) $\vec{E} = \frac{1}{4\pi \in_0}\frac{p}{r^3}(-\hat{j}) + \frac{1}{4\pi \in_0}\frac{p}{r^3}\hat{j} = 0 \Rightarrow Q \to 1$

(R) $\vec{E} = \frac{1}{4\pi \in_0}\frac{p}{r^3}(-\hat{j}) + \frac{1}{2\pi \in_0}\frac{p}{r^3}\hat{i} \Rightarrow R \to 4$

(S) $\vec{E} = \frac{1}{2\pi \in_0}\frac{p}{r^3}\hat{i} + \frac{1}{2\pi \in_0}\frac{p}{r^3}\hat{i} \Rightarrow S \to 5$

15.(A) **(P)** I and V are in same phase

$I_0 = \frac{300}{30} = 10A \quad \therefore \quad P \to 3$

(Q) $X_L = \omega L = 400 \times 100 \times 10^{-3} = 40\Omega$

$Z = \sqrt{R^2 + X_L^2} = 50\Omega$

$I_0 = \frac{300}{50} = 6A$

So $i = (6A)\sin(400t - 53°)$

I legs V $\quad \therefore \quad Q \to 5$

(R) $X_L = \omega L = 400 \times 25 \times 10^{-3} = 10\Omega$

$X_C = \frac{1}{\omega_C} = \frac{1}{400 \times 50 \times 10^{-6}} = 50\Omega$

$\therefore \quad z = \sqrt{30^2 + (50-10)^2} = 50\,\Omega$

$I_0 = \frac{300}{50} = 6A \Rightarrow i = (6A)\sin(400t + 53°)$

I leads V

$\therefore \quad R \to 2$

(S) $X_L = 400 \times 125 \times 10^{-3} = 50\Omega$

$X_C = \frac{1}{400 \times 50 \times 10^{-6}} = 50\Omega \quad \therefore \quad z = R = 60\,\Omega$

$I_0 = \frac{300}{60} = 5A \quad \Rightarrow \quad i = (5A)\sin 400t$

I and V are in same phase $\quad \therefore \quad S \to 1$

16.(C) **(P)** $E = \frac{-13.6Z^2}{n_1^2} - \left(\frac{-13.6Z^2}{n_2^2}\right)$

For $E \propto Z^2$ for transition of electron in hydrogen like atoms.

(Q) In case wavelength of $K_{\alpha} line$

$\frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\Rightarrow \quad \frac{hc}{\lambda} = hcR(Z-1)^2\left(\frac{3}{4}\right)$

$E \propto (Z-1)^2$

So, energy of characteristic X rays $\propto (Z-1)^2$

(R) If E be energy electrostatic intraction of Z protons in a nucleus then $E \propto Z(Z-1)$

(S) If E is independent of Z, then

It is binding energy per nucleon for stable nuclear having Z between 30 to 170

So, answer C

$P \to 5 \quad Q \to 1 \quad R \to 2 \ +57.4$

CHEMISTRY

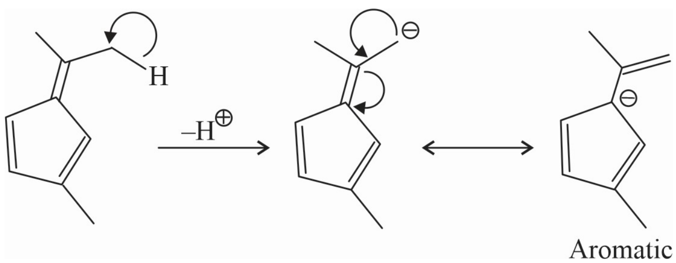
1.(A) $NH_4NO_2 \xrightarrow{60°-70°C} \underset{(X)}{N_2} + 2H_2O$; $NH_4NO_3 \xrightarrow{200°-250°C} \underset{(Y)}{N_2O} + 2H_2O$

2.(A)

Coordination entity	Wavelength of light absorbed (nm)
$[Co(NH_3)_5(Cl)]^{2+}$	535
$[Co(NH_3)_5(H_2O)]^{3+}$	500
$[Co(NH_3)_6]^{3+}$	475
$[Co(CN)_6]^{3-}$	310

3.(B) $2MnO_4^- + H_2O + I^- \rightarrow 2MnO_2 + 2HO^- + IO_3^-$

4.(B)



5.(BC) $Ne_2 \Rightarrow (\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2(\pi_{2p_x})^2(\pi_{2p_y})^2(\pi_{2p_x}^*)^2(\pi_{2p_y}^*)^2(\pi_{2p_z}^*)^2$

$$\text{Bond order} = \frac{N_{BMO} - N_{ABMO}}{2} = \frac{10-10}{2} = 0$$

$F_2 \Rightarrow (\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2(\pi_{2p_x})^2(\pi_{2p_y})^2(\pi_{2p_x}^*)^2(\pi_{2p_y}^*)^2$

Bond order of O_2^+ (2.5) > Bond order of O_2 (2)

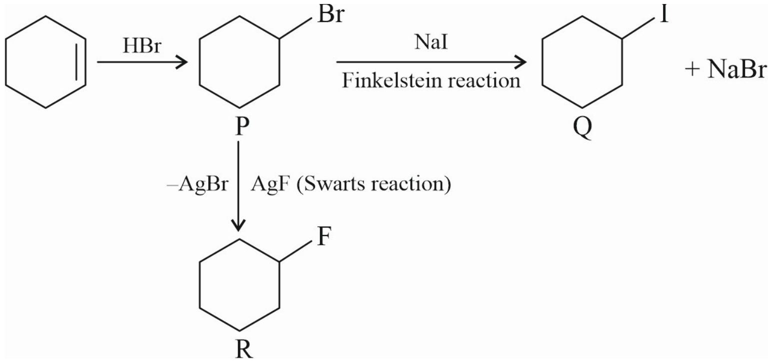
Bond energy of O_2^+ > Bond energy of O_2

Bond length of Li_2 (267 pm) > Bond length of B_2 (159 pm)

6.(AB)

La^{3+}	$4f^0$	Diamagnetic
Ce^{4+}	$4f^0$	Diamagnetic
Yb^{2+}	$4f^{14}$	Diamagnetic
Lu^{3+}	$4f^{14}$	Diamagnetic
La^{2+}	$5d^1$	Paramagnetic
Ce^{3+}	$4f^1$	Paramagnetic
Yb^{3+}	$4f^{13}$	Paramagnetic
Lu^{2+}	$4f^{14}5d^1$	Paramagnetic

7.(B)



$C-X$ bond length order	Q > P > R
$C-X$ bond enthalpy order	R > P > Q
S_N2 reactivity order	Q > P > R
pK_a order of HX	R > P > Q

8.(100) $Cr_2O_7^{2+} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

Number of faradays = 3 × Mole of Cr^{3+} produced

$$\frac{I \times 48.25 \times 60}{96500} = 3 \times 1$$

$$I = \frac{3 \times 1 \times 96500}{48.25 \times 60} = 100 \text{ amperes}$$

9.(2.24) $[H^+]_{Total} = [H^+]_{acid} + [H^+]_{H_2O}$

$$= \sqrt{K_a \times C + K_w}$$

$$= \sqrt{4.00 \times 10^{-11} \times 1.00 \times 10^{-3} + 10^{-14}} = \sqrt{5 \times 10^{-14}}$$

$$X \times 10^{-7} = 2.24 \times 10^{-7} \Rightarrow X = 2.24$$

10.(–7.1)

$$\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

$$pV_m + \frac{a}{V_m} - pb - \frac{ab}{V_m^2} = RT$$

$$pV_m^2 + a - \frac{ab}{V_m} = (RT + pb)V_m$$

$$pV_m^3 + aV_m - ab = (RT + pb)V_m^2$$

$$pV_m^3 - (RT + pb)V_m^2 + aV_m - ab = 0$$

Coefficient of $V_m^2 = -(RT + pb) = -[(0.082 \times 300) + (300 \times 0.060)] = -[24.6 + 18] = -42.6$

Coefficient of $V_m = a = 6$

$$-\frac{(RT + pb)}{a} = \frac{-42.6}{6} = -7.1$$

11.(–29.88)

$$2H_2O_{(l)} \rightarrow 2H_{2(g)}O_{2(g)}$$

$$w = -P_{ext}\Delta V = -P_{ext}[V_P - V_R] = -P_{ext}\left[\frac{n_T RT}{P_{ext}}\right] = -n_T RT$$

Mole of H_2 formed = mole of H_2O electrolysed = $\frac{144}{18} = 8$

Mole of O_2 formed = $\frac{1}{2} \times$ mole of H_2O electrolysed = 4

$$w = -12 \times 8.3 \times 300 = -29880 = -29.88 \text{ kJ}$$

12.(280)

$H_2N-(CH_2)_6-NH_2$ and $HOOC-(CH_2)_4-COOH$ are monomers of Nylon 6, 6.

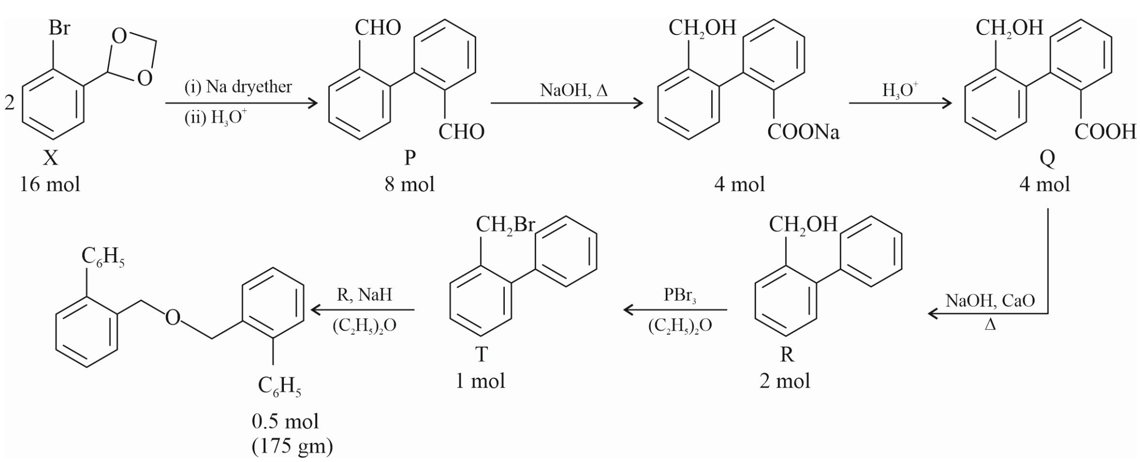
$H_2N-(CH_2)_6-NH_2$ gives positive carbylamine test.

$$C_6H_{16}N_2 + 20CuO \xrightarrow[CO_2]{\Delta} 6CO_2 + 8H_2O + N_2 + 20Cu$$

Mole of N_2 formed = mole of monomer $H_2N-(CH_2)_6-NH_2$ taken = 10

Amont of nitrogen gas formed = 10 × 2 × 14 = 280 g

13.(175)



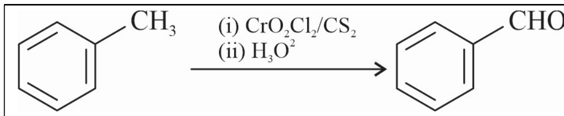
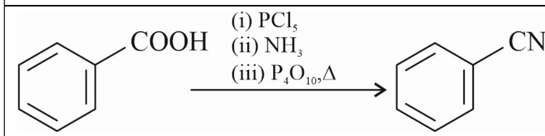
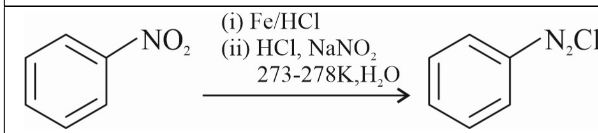
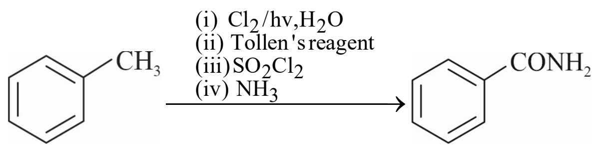
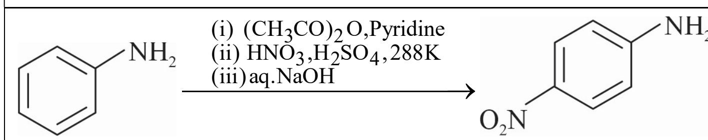
14.(A) **P → 3** : Passing H_2S in the presence of NH_4OH precipitates Mn^{2+}

Q → 4 : Passing $(NH_4)_2CO_3$ in the presence of NH_4OH precipitates Ba^{2+}

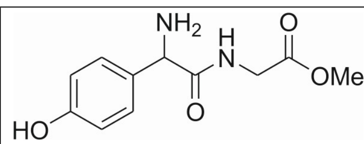
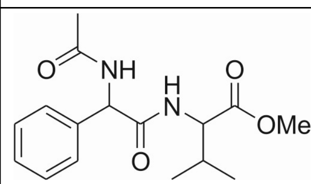
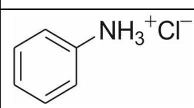
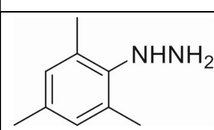
R → 2 : Passing NH_4OH in the presence of NH_4Cl precipitate Al^{3+}

S → 1 : Passing H_2S in the presence of dilute HCl precipitates Cu^{2+}

15.(B)

	Product is reactant of Cannizzaro reaction (S)-1
	Product is reactant of Stephen reaction (P)-2
	Product is reactant of Sandmeyer reaction (Q)-3
	Product is reactant of Hoffmann bromamide degradation reaction (R)-4
	Product is reaction of carbylamine reaction

16.(B)

P → 2		• Ninhydrin Test (amide) • Violet colour with $FeCl_3$ (Phenolic compound)
Q → 5		• Ninhydrin Test • After complete hydrolysis, gives ninhydrin test (amino acid) and does not give positive phthalein dye test (absence of phenolic OH group)
R → 1		• Reaction with phenyl diazonium salt gives yellow dye (Aniline yellow)
S → 3		• Reaction with glucose will give corresponding hydrazone.

MATHEMATICS

1.(C) $f(x)-g(x)\neq 0$

$$(a_3-b_3)x^3+(a_2-b_2)x^2+7x+(a_1-b_1)\neq 0$$

$$a_3=b_3 \qquad \ldots(1)$$

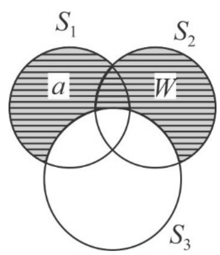
$$\underbrace{(a_2-b_2)x^2+7x+(a_1-b_1)\neq 0}_{\text{no. roots}}$$

Focus only on the coefficient of x^3

$$h(x)=\underbrace{a_3(x+1)^3+(x+1)^4-b_3}_{(a_3=b_3)[x^3-x^3]+4c_3-4c_3\cdot 2}(x+2)^3-(x+2)^4$$

$\Rightarrow \qquad 4-8=-4$

2.(A)



$$p(V)=P\left(\frac{S_1}{(S_2\cup S_3)'}\right)=\frac{1}{10}$$

$$P(S_1\cup S_2\cup S_3)=\frac{1}{2}$$

Let, $\quad P(S_1)=a$

$$\frac{a}{a+\frac{1}{2}}=\frac{1}{10}$$

$$20a=2a+1$$

$$18a=1$$

$$a=\frac{1}{18} \qquad\qquad P(W)=\frac{1}{12}$$

$$\frac{1}{2}-\left[\frac{1}{18}+\frac{1}{12}\right]=\frac{1}{2}-\frac{5}{36}=\frac{13}{36}$$

3.(C) The function

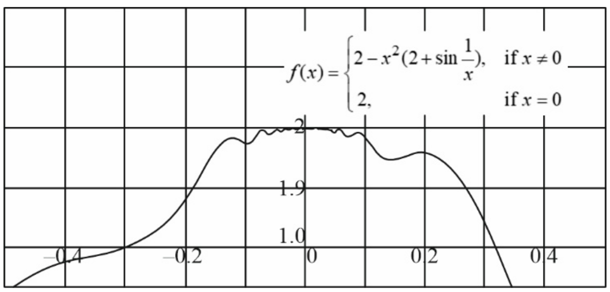
$$f(x)=\begin{cases} 2-x^2(2+\sin\dfrac{1}{x}), & \text{if } x\neq 0 \\ 2, & \text{if } x=0 \end{cases}$$

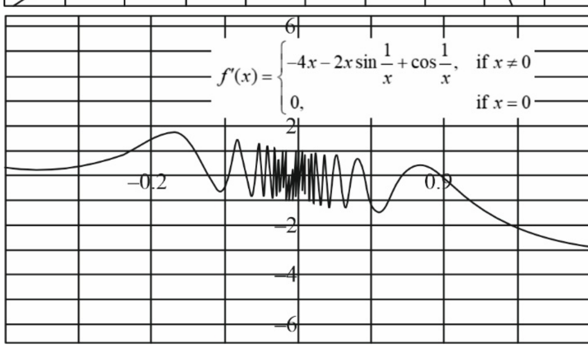
is continuous on $\mathbb{R}$. Since $x^2(2+\sin\dfrac{1}{x})$ is positive for all $x\neq 0$, then

$2>2-x^2(2+\sin\dfrac{1}{x})$. Therefore the function $y=f(x)$ has a local maximum at the point $x=0$. But it is neither increasing for all $x<0$ nor decreasing for all $x>0$ in any neighborhood of the point $x=0$. To show this we can find the derivative

$$f'(x)=-4x-2x\sin\frac{1}{x}+\cos\frac{1}{x};\quad x\neq 0.$$

The derivative takes both positive and negative values in any interval $(-\delta,0)\cup(0,\delta)$ and therefore the function is not monotone in any interval $(-\delta,0)\cup(0,\delta)$, where $\delta>0$.





4.(C) Let $Q=\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$

$QP=PQ$

$$P=\begin{bmatrix} 2a_1 & 2a_2 & 3a_3 \\ 2a_4 & 2a_5 & 3a_6 \\ 2a_7 & 2a_8 & 3a_9 \end{bmatrix}=\begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 2a_4 & 2a_5 & 2a_6 \\ 3a_7 & 3a_8 & 3a_9 \end{bmatrix}$$

So, from Equality

$$a_3=a_6=a_8=a_7=0$$

$$Q^{-1}=Q^T$$

So, $Q \cdot Q^T = I$

$$\begin{bmatrix} a_1 & a_2 & 0 \\ a_4 & a_5 & 0 \\ 0 & 0 & a_9 \end{bmatrix}\begin{bmatrix} a_1 & a_4 & 0 \\ a_2 & a_5 & 0 \\ 0 & 0 & a_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-a_1^2 + a_2^2 = 1$ ...(1)

$a_1 a_4 + a_2 a_5 = 0$ ...(2)

$a_4 a_1 + a_2 a_5 = 0$

$a_4^2 + a_5^2 = 1$ ...(3)

$a_9^2 = 1$ ...(4)

Determinant

$a_9(a_1 a_5 - a_4 a_2)$

$a_1 a_5 \neq a_4 a_2$

$a_9 = \pm 1$ from(1)

$a_1^2 = 1, \; a_2^2 = 0$

$a_2^2 = 0, \; a_5^2 = 1$

$a_2^2 = 0, \; a_2^2 = 1$

$a_4^2 = 1, \; a_5^2 = 0$

Total ways: $\underset{for\, a_9}{2} \times [8] = 16$

5.(AC) Equation of L_2: Direction $\begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(1) + k(1)$

$$L_2 : \frac{x-2}{1} = \frac{y+1}{-1} = \frac{2-3}{1} = \lambda$$

$P_1(\lambda + 2, -\lambda - 1, \lambda + 3)$

For Q: $2(\lambda + 2) + (-\lambda - 1) - 2(\lambda + 3) = 6$

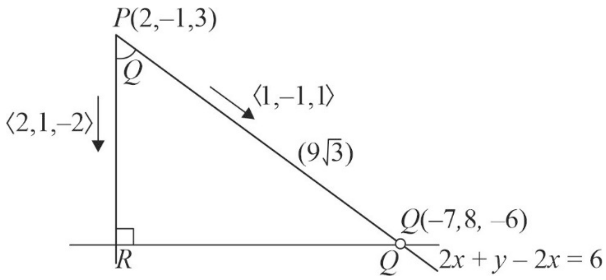
$2\lambda + 4 - \lambda - 1 - 2\lambda - 6 = 6$

$\lambda = -9$

$Q(-7, 8, -6) \quad P(2, -1, 3)$

(A) $PQ = \sqrt{9^2 + 9^2 + 9^3} = 9\sqrt{3}$

(B)



$$\cos\phi = \left|\frac{2-1-2}{\sqrt{9}\sqrt{3}}\right|$$

$$\cos\phi = \frac{1}{3\sqrt{3}} \qquad \ldots(1)$$

$$\frac{1}{3\sqrt{3}} = \frac{PR}{9\sqrt{3}}$$

$$PR = 3$$

$$QR = \sqrt{81\times 3 - 9} = \sqrt{234}$$

(C) Area of ΔPQR

$$\frac{1}{2}\times\sqrt{234}\times 3 = \frac{3}{2}\sqrt{234}$$

(D) Wrong, refer equation (1)

6.(AD) $f(x) = \begin{cases} \dfrac{2k+2}{2} = k+1, & n = 2k+1 \rightarrow n = \text{odd} \\ \dfrac{4-2k}{2} = 2-k & n = 2k \rightarrow n = \text{Even} \end{cases}$

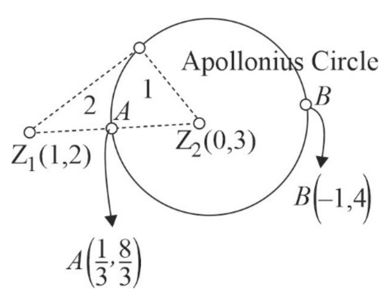
At $x = 2$ $\left.\begin{matrix} g(f(1)) = g(1) \\ f(f(1)) = g(1) \end{matrix}\right\rangle$ gof is not one-one

$g(f(x)) = g(\text{integer}) \rightarrow \text{not into}$ (as per option C we have done above)

Option (B) $f(9(x)) = f(\text{Natural } except\, 1)$ Hence it is one-one

B is wrong

7.(AD)



$$\text{Centre} = \left(\frac{\frac{1}{3} - 1}{2}, \frac{\frac{8}{3} + 4}{2} \right)$$

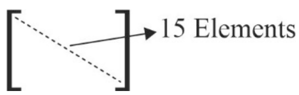
$$\left(-\frac{1}{3}, \frac{10}{3} \right)$$

$$r = \sqrt{\frac{4}{9} + \frac{4}{9}}$$

$$r = \frac{2}{3}\sqrt{2}$$

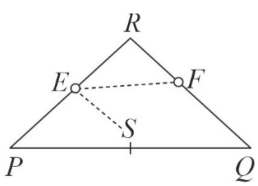
8.(105) Total = 36

Diagonal = 6 = 30



$$= {}^{15}C_2 = \frac{15 \times 14}{2} = 15 \times 7 = 105$$

9.(1.2)



p.v of $S \to s$

$P \to p$

$Q \to q$

$R \to r$

Length of $EF = \left| \frac{p+r}{2} - \frac{q+r}{2} \right| = \left| \frac{p-q}{2} \right|$ …(1)

Length of $ES = \left| \frac{p+r}{2} - S \right|$

For S, we have $\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}$

$$p - s + 5(q - s) + 6(r - s) = 0$$

$$\frac{p + 5q + 6r}{12} = s$$

$$= \left| \frac{p+r}{2} - \frac{p}{12} - \frac{5q}{12} - \frac{r}{2} \right|$$

$$= \left|\frac{p}{2} - \frac{p}{12} - \frac{5q}{12}\right|$$

$$= \left|\frac{5p}{12} - \frac{5q}{12}\right| = \frac{5}{12}\,|p-q| \qquad \ldots(2)$$

(1) $\div$ (2)

$$\frac{12}{10} = 1\cdot 2$$

10.(762) **Case-I:** O appears 2 times select 2 out of last 6 places in ${}^6C_2$ ways and place $\frac{1}{2}$ in any of remaining 5 places in $6C_1 \times 2^5 = 480$ ways

Case-II: 1 appears 2 times with 1st digit as 1 select 1 out of 6 places for 1 and place $\frac{0}{2}$ in rest 5 in $6C_1 \times 2^5 = 192$ ways

Case-III: 1 appears 2 times with 1st digit as 2 select 2 out of 6 places for 1 and place $\frac{0}{2}$ in rest 4 places ${}^6C_2 \times 2^4 = 240$ ways

Case-IV: '0' & '1' both 2 times.

1st digit is 1

$${}^6C_2 \times {}^4C_1 \times 1 = 60$$

Case-V: '0' & both 2 times.

1st digit is 2

$${}^6C_4 \times {}^4C_2 \times 1 = 90$$

By inclusion and exclusion

Case 1 + Case 2 + Case 3 – Case 4 – Case 5

480 + 192 + 240 – 60 – 90 = 762

11.(2.40)

L.H $$\frac{\frac{\alpha}{2}\left(\frac{1}{1-x^2}\right) - \beta x \sin x + \beta \cos x}{3\times 2} = 2$$

Using expansion

$$\frac{\frac{\alpha}{3}(1+x^2) - \beta \times x^2 + \beta\left(1 - \frac{x^2}{2}\right)}{3\times 2} = 2$$

$$\frac{\alpha}{2} + \beta = 0$$

$$\frac{\alpha}{2}-\beta-\frac{\beta}{2}=6$$

$$-\beta-\beta-\frac{\beta}{2}=6$$

$$-\frac{5\beta}{2}=6$$

$$\beta=\frac{-12}{5}$$

$$\alpha=\frac{24}{5}$$

$$\alpha+\beta=\frac{12}{5}=2.4$$

12.(96) $f(a_1+30d)=64f[a_1+24d]$

$$f(a_1)f(30d)=64f(a_1)f(24d)$$

$$K^{30d}=64K^{24d}\ \ set\ f(x)=k^x$$

$$k^{6d}=64 \qquad \ldots(1)$$

$$k^{a_1}+k^{a_1+d}+k^{a_1+2d}+....+k^{a_1+49d}=3(2^{25}+1)$$

$$\frac{k^{a_1}\left[k^{50d}-1\right]}{k^d-1}=3(2^{25}+1) \qquad \ldots(2)$$

Using (1) in (2)

$$k^{a_1}\left[2^{50}-1\right]=3(2^{25}+1)$$

$$k^{a_1}\left[2^{25}-1\right]\left[2^{25}+1\right]=3\left[2^{25}+1\right]$$

$$k^{a_1}=\frac{3}{2^{25}-1}$$

Now $\sum_{6}^{30}f(a_i)=K^{a_1+5d}+K^{a_1+6d}+....+K^{a_1+29d}$

$$=K^{a_1}.K^{5d}\frac{\left[K^{25d}-1\right]}{(K^d-1)}$$

$$=\frac{3}{(2^{25}-1)}\times 2^5\frac{\left[2^{25}-1\right]}{(2-1)}=32\times 3=96$$

13.(2) 3-L.D.E with $q = 0$

$$y_1 \cdot e^{-\int \frac{1-\cos 2x}{2} dx} = C_1$$

$$y_1 \cdot e^{-\left[\frac{x}{2} - \frac{\sin 24}{4}\right]} = C_1$$

$$(5)\ e^{-\left[\frac{1}{2} - \frac{\sin 2}{4}\right]} = C_1$$

$$y_2\, e^{-\int \frac{1+\cos 2x}{2} dx} = C_2$$

$$y_2\, e^{-\left[\frac{x}{2} + \frac{\sin 2x}{4}\right]} = C_2$$

$$\frac{1}{3} e^{-\left[\frac{1}{2} + \frac{\sin 2}{4}\right]} = C_2$$

$$Y_3\, e^{\int \left(\frac{2}{x^3} - 1\right) dx} = C_3$$

$$Y_3 \cdot e^{-\left[\frac{-1}{x^2} - x\right]} = C_3$$

$$\frac{3}{5e} e^2 = C_3$$

$$C_1 C_2 C_3 = 1$$

$$\lim_{x \to 0} \frac{e^{\frac{-1}{x^2}} + 2x}{(e^{3x})x} = \lim_{x \to 0^+} \frac{e^{\frac{-1}{x^2}} + 2x}{x} = 2$$

14.(C) Given median = 6

So, $f_1 + 5 = 9 = f_2 + 2 + 3 + 1$

$f_1 = 4,\ f_2 = 3$

(P) $7.4 + 9.3 = 28 + 27 = 55$

So, D is Rejected

Mean: $\dfrac{20 + 20 + 6 + 24 + 18 + 33 + 12}{19}$

$\Rightarrow \dfrac{133}{19} = 7$

$$\boxed{\begin{array}{l} 4 — 5 \\ 5 — f_1 \\ 6 — 1 \\ 8 — f_2 \\ 9 — 2 \\ 11 — 3 \\ 12 — 1 \\ f_1 + f_2 = 7 \end{array}}$$

Deviation by mean

3, 2, 1, 1, 2, 4, 5

$$\alpha = \frac{\overbrace{15+8}^{23}+\overbrace{1+3+4+12+5}^{25}}{19}$$

$19\alpha = 48$

$19\alpha = 18$ Q.3 (B-Rejected)

Deviation by Median $\rightarrow$ 2, 1, 0, 2, 3, 5, 6

$$\boxed{\begin{array}{l} 4 — 5 \\ 5 — f_1 \\ 6 — 1 \\ 8 — f_2 \\ 9 — 2 \\ 11 — 3 \\ 12 — 1 \\ f_1 + f_2 = 7 \end{array}}$$

$$\beta = \frac{\overbrace{10+4+0+6}^{20}+\overbrace{6+15+6}^{27}}{19}$$

$19\beta = 47$

$R \rightarrow 2$

Variance,

Deviation by Mean 3, 2, 1, 1, 2, 4, 5

Square of Deviation by Mean 9, 4, 1, 1, 4, 16, 25

$$\boxed{\begin{array}{l} 4 — 5 \\ 5 — f_1 \\ 6 — 1 \\ 8 — f_2 \\ 9 — 2 \\ 11 — 3 \\ 12 — 1 \\ f_1 + f_2 = 7 \end{array}}$$

$$\sigma^2 = \frac{45+16+1+3+8+48+25}{19}$$

$$19\sigma^2 = 146$$

15.(B) $f(1) = \left[\frac{10-\overbrace{45+60+35}}{n}\right] = \left[\frac{60}{n}\right]$

$$f(2) = \left[\frac{\overbrace{80-180}+120+35}{n}\right]$$

$$f(2) = \left[\frac{55}{n}\right]$$

From the option $n = 9$

$P \to 2$

$$g(x) = (2n^2 - 13n - 15)\ (3x^2 + 3) > 0$$

$$2n^2 - 15n + 2n - 15 > 0$$

$$(n+1)(2n-15) < 0$$

$$n < -1,\ n > \frac{15}{2}$$

$$n = 8$$

16.(A) $\vec{v} = (\vec{v} \times \vec{w})$

$$\vec{u} \times \vec{v} = (\vec{v} \times \vec{w}) \times \vec{v}$$

$$\vec{w} = (\vec{v} \cdot \vec{v})\vec{w} - (\vec{w} \cdot \vec{v})\vec{v}(0)$$

$$\vec{w} = |\vec{v}|^2 \cdot \vec{w} \qquad \ldots(1)$$

D-Rejected

From, Q, R, 5 α is non-zero

So, α, β, γ cannot be All zero.

$D = 0$

$$\begin{vmatrix} -t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{vmatrix} = 0$$

$-t(t^2-1)-1(-t-1)+1(1+t)=0$

$-t^3+t+t+1+t+1=0$

$-t^3+3t+2=0$

$t^3-3t-2=0$

$t^3+t^2-t^2-t-2t-2=0$

$(t+1)\left[t^2-t-2\right]=0$

$(t+1)\left[t^2-2t+t-2\right]=0$

$(t+1)^2(t-2)=0$

$t=-1,\ t=2$

$-t\alpha+\beta+\gamma=0,\ \alpha-t\beta+\gamma=0$ and

$t=-1 \rightarrow \alpha+\beta+\gamma=0$

$t=2 \rightarrow \beta+\gamma=2\alpha$

$2\beta=2\alpha$

$\alpha=\beta$

$\alpha+\gamma=2\beta$

$\alpha+\beta=2\gamma$

$-$

$\gamma-\beta=2\beta-2\gamma$

$\beta-\gamma$

$\alpha=\beta=\gamma$

$\vec{u}\times\vec{v}=\vec{w}$

$|u||v|=|w|$

$(\alpha^2+\beta^2+\gamma^2)1=6$

$\alpha^2+\beta^2+\gamma^2=6$

Option B : $\alpha=\sqrt{3}$ possible when $t=-1$

$\vec{u}\cdot\vec{w}=0 \rightarrow \alpha+\beta=2\gamma$

$\alpha+\beta-t\gamma=0$

$2\gamma-t\gamma=0$

$\gamma(2-t)=0$

$y=0$

$t\neq 2$

Option C:

$$\begin{cases} \alpha = \sqrt{3} \\ \alpha + \beta + \gamma = 0 \end{cases}$$

$$\sqrt{3} + \beta + 0 = 0$$

$$\beta = -\sqrt{3}$$

$$(\beta + \gamma)^2$$

$$(-\sqrt{3} + 0) \qquad \ldots(3)$$

Option D: $\alpha = \sqrt{2}$

$$\alpha^2 + \beta^2 + \gamma^2 = 6$$

$$\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 = 6$$

Means, $t = 2$

$(t + 3) = 5$