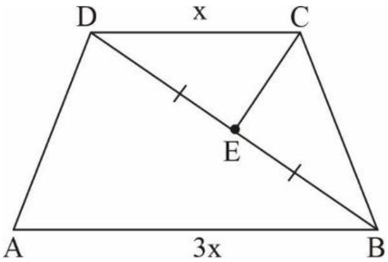
IOQM | SOLUTIONS

1.(8)



$$\frac{ar(\text{ABCD})}{ar(\text{CDE})}=n$$

$$\frac{ar(\Delta\text{ABD})}{ar(\Delta\text{CBD})}=\frac{\frac{1}{2}\times h\times 3x}{\frac{1}{2}\times h\times x}=3$$

also $ar(\Delta\text{CBD})=2(ar(\Delta\text{CDE}))$

$\Rightarrow \quad \frac{ar(\Delta\text{ABD})}{2ar(\Delta\text{CDE})}=3 \quad \Rightarrow \quad ar(\Delta\text{ABD})=6ar(\Delta\text{CDE}) \quad \therefore \quad ar(\text{ABCD})=8ar(\Delta\text{CDE}) \Rightarrow = 8$

2.(64) $5b^2+0b+3b^0=3(b+2)^2+0\times(b+2)^1+5\times b^0$

$5b^2+3=3(bf_2^2)^2+5b$

$5b^2+3=3b^2+12+12b+5$

$2b^2-12b-14=0$

$b^2-6b+7=0$

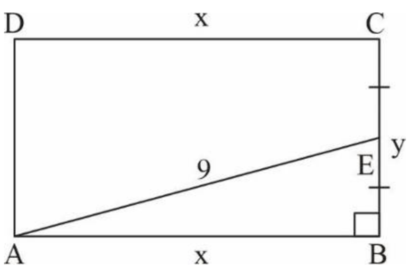
$(b-7)(b+1)=0,\ b=7$

$N=5\times 7^2+3=248 \rightarrow$ Product of digits $=2\times 4\times 8=64$

3.(99) $\sum_{k=1}^{n}\frac{2k+1}{k^2(k+1)^2}=0.9999 \Rightarrow \quad \sum_{k=1}^{N}\left[\frac{1}{k^2}-\frac{1}{(k+1)^2}\right]=\frac{9999}{10000}$

$\Rightarrow \quad 1-\frac{9999}{10000}=\frac{1}{(k+1)^2} \Rightarrow k+1=10 \Rightarrow \quad k=99$

4.(19)



(1) $\quad 2x+y=20$

(2) $\quad x^2+\frac{y^2}{4}=81$

$\Rightarrow \quad 4x^2 + y^2 = 324$

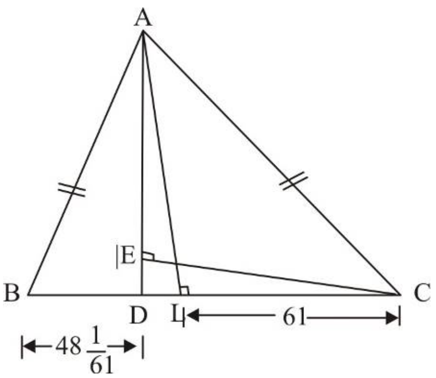
Squaring (1) $\quad 4x^2 + y^2 + 4xy = (20)^2 \quad \Rightarrow \quad 324 + 4xy = 400 \quad \Rightarrow \quad xy = 19$

5.(18) $\left||x| - 2020\right| < 5 \quad \Rightarrow \quad -5 < |x| - 2020 < 5$

$\Rightarrow \quad 2015 < |x| < 2025 \quad \Rightarrow \quad x \in (-2025, 2015) \cup (2015, 2025) \quad \Rightarrow$ integer values are 18

6.(15) $2^5 \times 3^6 \times 4^3 \times 5^3 \times 6^7 \quad \Rightarrow \quad 2^{18} \times 3^{13} \times 5^3 \quad \Rightarrow$ multiply by $3 \times 5 = 15$

7.(25)



$BC = \frac{6650}{61}$

$LD = CD - LL = 61 - \frac{BC}{2} = 61 - \frac{3325}{61} = \frac{396}{61}$

$ALD \sim CED$

$\frac{AD}{CD} = \frac{LD}{ED} \Rightarrow AD = 61 \times \frac{396}{61} \times \frac{1}{11}$

$AD = 36$

So, $AE = 36 - 11 = 25$

8.(15) $k, k+1, k+2, 3k, k+3$

k must be either 1, 2 or 3 $\quad \because 3k$ must be then 10 so numbers can be

1, 2, 3, 3, 6

Or 2, 3, 4, 6, 5

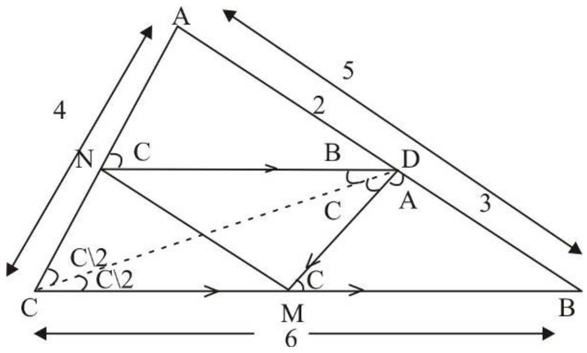
Or 3, 4, 5, 9, 6

Out of these 3 only 34596 (i) square of 186

So $m = 186$

And sum of digits of $m_{13} = 15$

9.(2)



$$\frac{AD}{BD} = \frac{4}{6} = \frac{2}{3}$$ (angle bisector prop)

$AD = 2, BD = 3$

And $BM = \frac{13}{5}, CM = \frac{12}{5} \left(\because By\, BPT\right)$

$DBM \sim ABC$

$$\frac{BM}{BC} = \frac{DM}{AC} \Rightarrow \frac{\frac{18}{5}}{6} = \frac{DM}{4}$$

$$AN = \frac{8}{5}, NC = \frac{12}{5}$$

$ANO \sim ACB$

$$\frac{AN}{AC} = \frac{ND}{BC} \Rightarrow \frac{8/5}{4} = \frac{ND}{6}$$

$$ND = \frac{12}{5}$$

In ΔDNM

$$\cos C = \frac{DN^2 + DM^2 - MN^2}{2\Delta NDM} \quad \text{.........(i)}$$

In ΔACB

$$\cos C = \frac{BC^2 + AC^2 - AB^2}{2BC.AC} \text{........(ii)}$$

Equal (i) and (ii)

$$\frac{\left(\frac{12}{5}\right)^2 + \left(\frac{12}{5}\right)^2 - MN^2}{2.\frac{12}{5}.\frac{12}{5}} = \frac{(6)^2 + (4)^2 - (5)^2}{2.6.4} \qquad \Rightarrow 2\left(\frac{144}{25}\right) - MN^2 = 27 \times \frac{6}{25}$$

$$MN^2 = \frac{286}{25} - \frac{162}{25} = \frac{126}{25} \quad \therefore \quad P = 123, q = 25$$

$|P - Q| = 101$

Sum of digit $|P - Q| = 2$

10.(40) say $0 \le x_i \le 100$

$$<\bar{x}> = \frac{\sum x_i}{5}$$

And $x_1 \le x_2 \le x_3 \le x_4 \le x_5$

$$\text{Difference } = \left| \frac{\sum x_i}{5} - x_3 \right| = \left| \frac{x_1 + x_2 + x_3 + x_4 - 4x_3}{5} \right| = \left| \frac{(x_1 - x_3) + (x_2 - x_3) + (x_4 - x_3) + (x_5 - x_3)}{5} \right|$$

To maximize $x_1 = x_3 \; and \; x_2 = x_3$

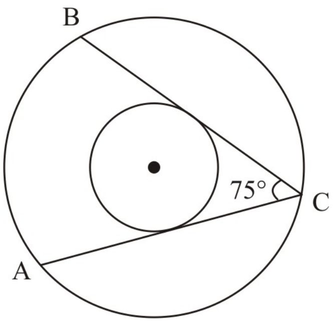
As x_4 and $x_5 = 100$

And $(x_4 - x_3) \; and \; (x_5 - x_3)$ should be maximum if $x_3 = 0$

$$\text{Difference } = \frac{200}{5} = 40$$

11.(24)

12.(24)



We will return to point A if $75(k_1) = 360(k_2)$ and we are looking for the least value of k_1

$\Rightarrow k_1 = 24$

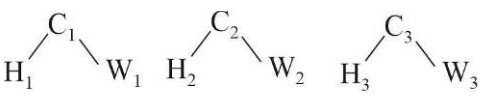
13.(6) $\left| 2^n + 5^n - 65 \right|$ is a perfect square .

The only possible values of n are 2,4

Hence the sum is 6.

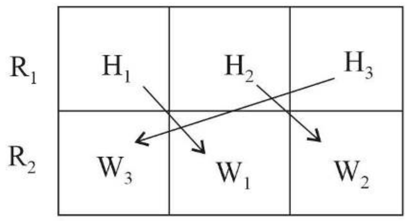
14.(20) $a \times b \times c \times d \times e = 2^2 \times 3 \times 5^3 \times 11 \times 13.$
$= 5 \times 11 \times 13 \times 15 \times 20.$

15.(96)



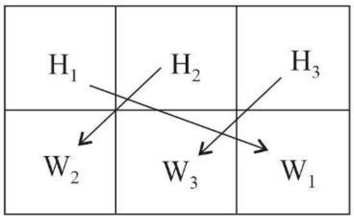
$6 \times 4 \times 2 = 48$ cases

Case I



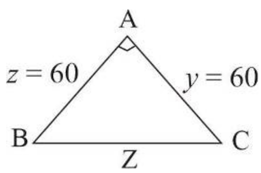
$6 \times 4 \times 2 = 48$ cases

Case II



$\therefore$ Total = 48 + 48 + 96 cases.

16.(87)



$$\left(y-x\right)=\frac{2\Delta\left(y-x\right)}{xy}$$

$\Delta=\frac{1}{2}xy.$ $\Rightarrow$ $\angle A=90°$ $\therefore$ largest side = $Z=\sqrt{x^2+y^2}=\sqrt{60^2+63^2}=87$

17.(30) 4 positive factors

2 digit numbers

$10a+b=a_1a_2a_3a_4=2^{k_1}3^{k_2}\left(5\right)^{k_3}\left(7\right)^{k_4}$

For $k_1=3$ $\left(1,2,4,8\times\right)$

$k_2=3$ $\left(1,3,9,27\right)$

$k_3=2\left(1,5,25\right)$ $\nearrow 50$, $\searrow 75$ not possible

$k_4=1,7,49 \nearrow 98,2$ not possible

$k_3=1,$ $1,5$

$\rightarrow 7,35$

$\rightarrow 11,55$

$\rightarrow 13,65$ $k_1=1,$ $1,2$ $3,6\times$

$\rightarrow 17,85$ 5, 10

$k_4 = 1$	1,7		7,14
		11,77	11, 22
		13,91	13, 26
			17,34
		2, 6 ×	19,38
$k_2 = 1$	1,3	5,15 ×	23,46
		7,21	29,58
		13,39	31,62
		17,51	37,74
		19,57	41,82
		23,69	43,86
		29,87	47,94
		31,93	49,98
		1,3,11,33	

30 such numbers possible.

18.(80) $\sum_{k=1}^{40}\left(\sqrt{1+\frac{1}{k^2}+\frac{1}{(k+1)^2}}\right) = a+\frac{b}{c}$

$$T_k = \sqrt{\left(\frac{k^2+k+1}{k(k+1)}\right)^2} = \frac{k^2+k+1}{k(k+1)} = 1+\frac{1}{k(k+1)}$$

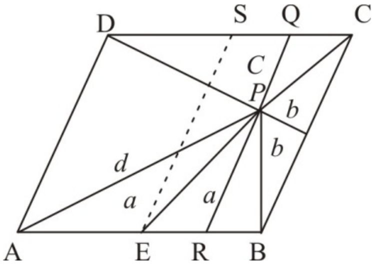
$$t_k = \frac{1}{k} - \frac{1}{k+1}$$

$$s_k = 1 - \frac{1}{41}$$

$$s = 40 + 1 - \frac{1}{41} = 40 + \frac{40}{41}$$

$$a + b = 80$$

19.(40) Area of □$APQ\Delta$



$= 2(area\ 7\ \Delta APD)$ (between some parallel line AD and PQ)

Area of $\square PBCQ = 2\,(area\ 7\ \Delta BPC)$ (between same parallel line PQ and BC)

Area o 7□$PBCQ = 2$ (area 7 Δ BPC) (between same parallel line PQ and BC)

Area 7 $\Delta APD +$ area 7 $\Delta BPC = \frac{1}{2}$(area 7 $\square ABC\,D$)

$d + 2b = 50$ … (i) $\rightarrow d = 50 - 2b$

Similarly $a + 2b = 25$ … (ii) $\rightarrow a = 25 - 2b$

$b + c = 25$ … (iii) $c = 25 - b$

$a + d + c = 75$ … (iv)

$25 - 2b + 50 - 2b + 25 - b = 75$

$100 - 5b = 75 \quad 5b = 25$

$b = 5$

Area 7 $\Delta APB = 50 - 2 \times 5 = 40$ unit maximum area.

20.(0) Let n women $\rightarrow$ 45 hours

Work done by one women $= \frac{1}{45n}$ per hour

Let work was completed in t hour when working one by one x = internals

$$\frac{1}{45n}\left[t + \left(t - \frac{t}{n}\right) + \left(t - \frac{2t}{n}\right) + \ldots t - \frac{(n+1)t}{n}\right] = 1$$

$$\frac{t}{45n}\left[n - \frac{1}{n}\left(1 + 2 + \ldots (n+1)\right)\right] = 1$$

$$\frac{t}{45n}\left[n - \frac{1}{n} \times \frac{(n-1)}{21}\right] = 1$$

$$\frac{t}{45n}\left[\frac{2n - (n-1)}{2}\right] = 1$$

From given condition

$$t = 5\left(t - \frac{(n-1)t}{n}\right)$$

$$\frac{1}{5} = 1 - \frac{n-1}{n} = 1 - 1 + \frac{1}{n} \qquad \boxed{n = 5}$$

$$t\left[n + 1\right] = 90n$$

$$t = \frac{90n}{n+1} = \frac{90 \times 5}{6} = 75 \text{ hours}$$

21.(35) N every year

↙↓↘

A B C

$$\frac{N}{2} \quad b\left(\frac{3N}{7} - 1\right) < \left(\frac{N}{14} + 1\right) \qquad b + c = \frac{N}{2}$$

$$\frac{3N}{7} \qquad b - 1\left(\frac{3N}{7} - 2\right) \quad 2C\left(\frac{N}{7} + 2\right) \qquad \frac{3N}{7} + b + 2c - 1 = N$$

$$b + 2c - 1 = \frac{4N}{7}$$

$$b+2c=\frac{4N}{7}+1$$

$$c+\frac{N}{2}=\frac{4N}{7}+1$$

$$c=\frac{4N}{7}-\frac{N}{2}+1 \;=\frac{N}{14}+1$$

$$b+\frac{N}{14}+1=\frac{N}{2}$$

$$b=\frac{N}{2}-\frac{N}{14}-1=\frac{3N}{7}-1$$

Diff of age will be constant.

$A\rightarrow B$

$$\frac{N}{2}-\left(\frac{3N}{2}-1\right)=\left(\frac{N}{14}+1\right)$$

$A-C$

$$\left(\frac{N}{2}-\frac{N}{14}-1\right)=\frac{6N}{14}-1=\frac{3N}{7}-1$$

$$\frac{N}{2}-\left(\frac{3N}{7}-1\right)=\left(\frac{N}{14}+1\right)$$

$$\frac{3N}{7}-\left(\frac{3N}{7}-2\right)=2$$

$$\frac{\frac{N}{14}+1}{2}=\frac{\frac{3N}{7}-1}{2\left(\frac{N}{7}-1\right)}$$

$$\left(\frac{N}{2}-\frac{N}{14}-1\right)=\frac{6N}{14}-1=\frac{3N}{7}-1$$

$$\frac{3N}{7}-\left(\frac{3N}{7}-2\right)=2$$

$$\frac{3N}{7}-\frac{N}{7}-2=\frac{2N}{7}-2$$

$$\frac{\frac{N}{14}+1}{2}=\frac{\frac{3N}{7}-1}{2\left(\frac{N}{7}-1\right)}$$

$$\left(\frac{N}{14}+1\right)\left(\frac{N}{7}-1\right)=\frac{3N}{7}-1$$

Let $\frac{N}{14}=k$

$$(k+1)(2k-1)=6k-1$$

$$2k^2+k-1=6k-1$$

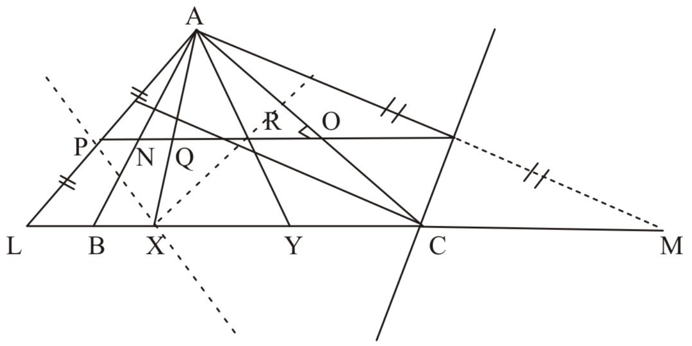
$$2k^2-5k=0$$

$k(2k-5)=0$

$k=\frac{5}{2}$

$\frac{N}{14}=\frac{5}{2} \rightarrow N=35$

22.(84)



P is middle point $P'Q$ AL

Q is middle point AX

R is middle point AY

S is middle point AM

Hence, $PQ \parallel LX, QR \parallel XY, RS \parallel YM$ (By mid-point theorem)

And N is also mid point of AB

O is also mid point of AC

APBR and AQCS are rectangle

So, $PR=AB=13$ and $AC=QS=14$

$NR=\frac{AB}{2}=6.5$ $OQ=\frac{QS}{2}=7$

$NQ=NR-6$ $RO=OQ-6$

$NQ=.5$ $RO=1$

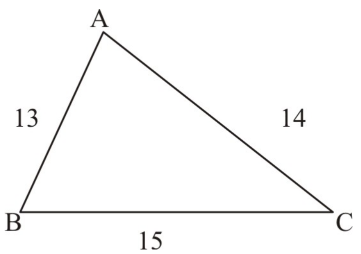
In ΔABX, $BX=2NQ=1$

In AYC, $YC=2.RO=2$ {by mid pt theorem}

And In ΔAXY

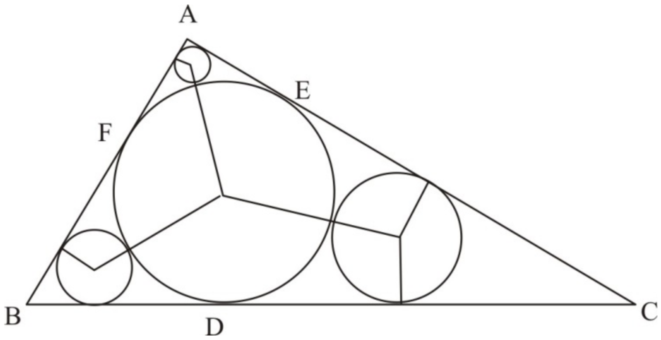
$XY=2PQ=12$ {by mid point theorem)

Now ΔABC is



Area $=\sqrt{21(6)(7)(8)}=84$

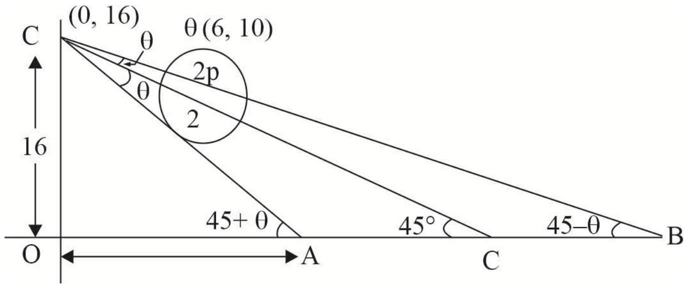
23.(74) $r=\sqrt{r_A r_B}+\sqrt{r_B r_C}+\sqrt{r_C\cdot r_A}=20+30+24=74$.



24.(21) Slope of $PC=\frac{-10}{10}=-1$

AB = OB – OA

$=\frac{16}{\tan(45+\theta)}$



$=\frac{16}{\tan(45+\theta)}-\frac{16}{\tan(45+\theta)}$

$CP=\sqrt{36+36}=6\sqrt{2}$ $\qquad \tan\theta=\frac{2}{\sqrt{68}}=\frac{1}{\sqrt{17}}$

$AB=16\left[\frac{1}{\tan 145+\theta}-\frac{1}{\tan 145+\theta}\right]$

$AB=16\left[\frac{1}{\tan 145+\theta}-\frac{1}{\tan 145+\theta}\right]$

$\tan(45-\theta)=\frac{\sqrt{17}-1}{\sqrt{17}+1}$

$AB=16\left[\frac{\sqrt{17}+1}{\sqrt{17}-1}-\frac{\sqrt{17}-1}{\sqrt{17}+1}\right]=16\left[\frac{17+1+2\sqrt{17}-\left[17+1-2\sqrt{17}\right]}{16}\right]$

$=2\sqrt{17}$

Hence = 21

25.(56) The given problem is

$<91><120><143><180><N>$

$=91 \quad .\ 120. \quad 143 \quad 180 \quad \text{N}$

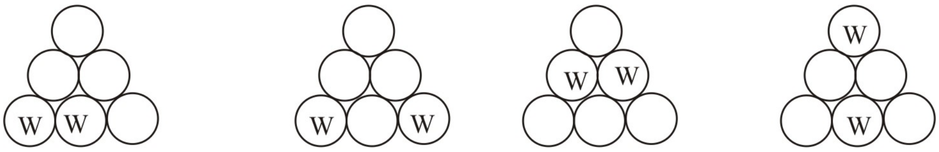
$\Rightarrow 100.121.144.169<N>=91.120.143.180N$

$\Rightarrow 22<N>=21N\Rightarrow 21|<N>$

But $<N>$ is a perfect square so $441<N>$

We had be lucky $<N>=441$ satisfies the equation put $<N>=441$ get $N=462$ we are lucky indeed sum of squares of digits $=56$

26.(18)



I II III IV

To eliminate the prossibility of repetition, let's think of the whole scenario w.r.t. one colour.

$I \rightarrow \frac{4!}{2!2!}=6$ cases

In II, III, IV we have 4 ways to colour.

Hence answer = 18

27.(87) The bug can travel along parallel vertical and I horizontal lines except for the 12 points. So the total number of points $=11\times 9-12=87$

28.(93) All numbers except power of 2.
Say if a number is odd $=2k=k+k+1$
Say if a number is even but has an odd divisior
Then $k(k+1)+..........+(k+r+1)=a$

$rk+(1+2+3+.......+r-1)=a \Rightarrow \quad rk+\frac{r(r)}{2}=a \Rightarrow r\left(k+\frac{r-1}{2}\right)=a$

Now $r-1=$ even $\Rightarrow r=$ odd.
So, for every a for which there is an odd divisor we have an answer $=100-(7)=93$

29.(99) Say $a-b=K>0\, as\, c>0$

$$a+b+c=b+k+b+\frac{b(b+k)}{k}=3b+k+\frac{b^2}{k}\geq 5b \text{ (using A.M. } - \text{ G.M.)}$$

$\Rightarrow b\leq 19$

$as\, a+b+c\leq 99$

Try $b=19,$ the max value of $a+b+c$ is 95

Try $b=18,$ the max value of $a+b+c=99$ for $a=27$ as $c=44$

30. Case – 1: $C-km-1=0$

$\Rightarrow \quad C-km-1\leq 1 \quad \Rightarrow \quad (k-2)=C(c-km-1)\leq -11 \quad \Rightarrow \quad k\leq -9$ {impossible}

Case – 2: $C-km-1>0$

$\Rightarrow \quad C\geq km+2$ …. (i)

$\Rightarrow \quad C\geq km+2$ … (ii)

$k-2\geq C$ … (2)

$\Rightarrow \quad km+2\leq k-2$

Impossible

$\Rightarrow \quad C=km+1$

Let $C=a+1$

[redacted]

Given $C | b |$

$\Rightarrow \quad b = mc + 1, m \in z^+$

Also $b | c^2 - c + 2$

$\Rightarrow \quad c^2 - c + 2 \geq 100 \Rightarrow \quad c \geq 11$

Also, $c^2 - c + 2 = kb, k \in z^+ \quad \Rightarrow \quad c^2 - c + 2 = kmc + k \quad \Rightarrow \quad c\left[c - km - 1\right] = k - 2$

Now, $c - km - 1 \in z$

AND $c \geq 11 \quad \Rightarrow \quad c = 2m + 1 \quad \Rightarrow \quad a = 2m$

AND $b = mc + 1 = 2m^2 + m + 1$

Also, $b \in \left[100, 999\right]$

Gives us $m \in [7, 22]$

16 possible values.